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Stochastic optimal operation of reservoirs based on copula functions

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ABSTRACT

Stochastic dynamic programming (SDP) has been widely used to derive operating policies for reservoirs considering streamflow uncertainties. In SDP, there is a need to calculate the transition probability matrix more accurately and efficiently in order to improve the economic benefit of reservoir operation. In this study, we proposed a stochastic optimization model for hydropower generation reservoirs, in which 1) the transition probability matrix was calculated based on copula functions; and 2) the value function of the last period was calculated by stepwise iteration. Firstly, the marginal distribution of stochastic inflow in each period was built and the joint distributions of adjacent periods were obtained using the three members of the Archimedean copulas, based on which the conditional probability formula was derived. Then, the value in the last period was calculated by a simple recursive equation with the proposed stepwise iteration method and the value function was fitted with a linear regression model. These improvements were incorporated into the classic SDP and applied to the case study in Ertan reservoir, China. The results show that the transition probability matrix can be more easily and accurately obtained by the proposed copula function based method than conventional methods based on the observed or synthetic streamflow series, and the reservoir operation benefit can also be increased.

1. Introduction

The reservoir hydropower operation is generally formulated as a nonlinear, stochastic optimization problem. To guide the reservoir operation efficiently, the reservoir operators usually predefine an operating policy in the design stage and use it in the real-time operation. According to the methods used to deal with stochastic inflow, the operating policy can be obtained by 1) implicit stochastic optimization (Chandramouli and Deka, 2005; Chandramouli and Raman, 2001; Mehta and Jain, 2009; Mousavi et al., 2005; Panigrahi and Mujumdar, 2000; Willis et al., 1984; Young, 1967), which can convert a stochastic optimization problem into a deterministic one to operate the reservoir under several equally likely inflow scenarios and then extract the operating rules from the resulting set of optimal operating data; 2) parameterization-simulation-optimization (Koutsoyiannis and Economou, 2003; Koutsoyiannis et al., 2010; Momtahen and Dariane, 2007; Nalbantis and Koutsoyiannis, 1997; Oliveira and Loucks, 1997; Tan et al., 2017a), which first predefines a shape for the rule curve based on some parameters and then use heuristic strategies to look for the combination of parameters that provide the best reservoir operating performance under possible inflow scenarios; and 3) explicit stochastic optimization (Chou et al., 2016; Harboe, 1993), which converts the inflow with different probabilities directly into the optimization problem.

Stochastic dynamic programming (SDP) is one of the most popular explicit stochastic optimization methods (Celeste and Billib, 2009). Previous studies on SDP have focused mainly on the inflow uncertainties and the curse of dimensionality. The classical SDP model (Loucks et al., 1981) does not take into account the forecast information in addressing the inflow uncertainties in reservoir operation. Given the significant effect of the forecast uncertainty on the reservoir operation, it is explicitly incorporated into some reservoir operation models (Karamouz and Vasiliadis, 1992; Mujumdar and Nirmala, 2007; Stedinger et al., 1984; Xu et al., 2014; Kim and Palmer, 1997). Karamouz and Vasiliadis (1992) proposed a Bayesian SDP (BSDP) model, in which a Bayesian approach was incorporated within the SDP formulation. BSDP is different from the classical SDP in choosing state variables. Bayesian Decision Theory can easily incorporate new information by updating prior probabilities to posterior probabilities, thus reducing the effect of natural and forecast uncertainties on the model. Kim and Palmer (1997) proposed a BSDP model to investigate the value

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of seasonal inflow forecasts in hydropower generation, based on which the monthly operating policies were derived for Skagit Hydropower System. Mujumdar and Nirmala (2007) used the BSDP model to derive an operating policy for a multi-reservoir hydropower generation system. Xu et al. (2014) proposed a new two-stage BSDP (TS-BSDP) model for real-time operation of cascaded hydropower systems to handle varying uncertainty of flow forecasts from Quantitative Precipitation Forecasts.

In the last decades, a number of aggregation-disaggregation models have been proposed to solve the curse of dimensionality. Mujumdar and Nirmala (2007) aggregated individual inflows of reservoirs in a cascaded hydropower system and used a BSDP model to represent the forecast uncertainty of the aggregate inflow. Tejada-Guibert et al. (2010) and Mujumdar and Nirmala (2007) proposed SDP models in which only the inflows of individual reservoirs were aggregated, and the storage of individual reservoirs and the aggregate flow were taken as state variables. Xu et al. (2014) proposed a TS-BSDP model with aggregate flow and storage of individual reservoirs as state variables to reduce the complexity of the optimal problem. The curse of dimensionality can also be avoided by approximating the value function by the iteration method (Cervellera et al., 2006; Drouin et al., 1996; Gal, 1979; Lee and Lee, 2006).

Although the previous researches about SDP have made great improvements. There are two aspects the previous study does not pay enough attention. On the one hand, the transition probability matrix is usually based on observed or synthetic streamflow series in which the probability is replaced with the frequency. Consequently, the precision of the transition probability matrix depends critically on the representativeness and size of the sample or the reliability of the synthetic streamflow series. The transition probability matrix can be distorted in the case that the observation occurs by chance or the synthetic streamflow series cannot well characterize the original data. On the other hand, the value function in the last stage is ignored in most previous studies. That is to say, there is no difference in future value no matter where the ending water storage of the last period is. This is obviously conflicting with the actual situation. As we all know, different carryover storages will bring different future benefits.

The copula function has been widely used in hydrology and water resources fields due to its ability to set up the joint distribution of multiple variables (Favre et al., 2004; Jenq-Tzong et al., 2010; Karmakar and Simonovic, 2009; Singh and Zhang, 2006; Reddy and Ganguli, 2012; Tan et al., 2017b). In this study, a new method based on copula function to calculate the transition probability matrix was proposed and the results were compared with the conventional statistical method. Meanwhile, the value in the last stage was calculated from a simple recursive equation by stepwise iteration, and the value function was fitted with a linear regression model. These improvements were incorporated into the classic SDP and then applied to guide the hydropower operation of Ertan reservoir, China. The next section introduces the proposed method. A case study is presented in Section 3, and the conclusions are drawn in Section 4.

2. Methodology

2.1. Development of recursive equation

2.1.1. Objective function and constraints

This study focuses on the reservoir whose main purpose is to maximize the total hydropower generation. Thus, the objective function is as follows:

$$f(s_{t-1}, q_t) = \max\left\{\sum_{t=1}^{T} E_{q_t}(B_t(s_{t-1}, q_t, s_t))\Delta t\right\} \tag{1}$$

where $f(s_{t-1}, q_t)$ is the maximum expected hydropower generation from period t to the entire planning horizon (T), in which t = 1, 2, ... T; $B_t(s_{t-1}, q_t, s_t)$ is the hydropower output in period t, in which s_{t-1} and s_t are the beginning and ending storages, and q_t is the inflow; E_{q_t} is the expectation operator; and Δt is the decision interval.

The following constraints should be included in the model:

(1) Water balance constraint.

$$s_t = s_{t-1} + (q_t - r_t)\Delta t \tag{2}$$

(2) Water storage constraint.

$$s_{t,\min} \leqslant s_t \leqslant s_{t,\max} \tag{3}$$

(3) Flow constraint.

$$r_{t,\min} \leqslant r_t \leqslant r_{t,\max}, \ Q_t \leqslant Q_{\max} \tag{4}$$

(4) Hydropower output constraint.

$$N_t \leqslant N_{\max} \tag{5}$$

(5) Relationship curves between reservoir water storage and its water level.

$$Z_t = f(V_t) \text{ and } V_t = f^{-1}(Z_t) \tag{6}$$

(6) Relationship curves between tail water level and its generation discharge.

$$Q_t = g(Z_{dr,t}) \text{ and } Z_{dr,t} = g^{-1}(Q_t) \tag{7}$$

where

r_t	The water release of the reservoir in period t, m^3/s
$r_{t,\max}, r_{t,\min}$	The upper and lower limits of the reservoir release in period t, m^3/s
$s_{t,\max}, s_{t,\min}$	The upper and lower limits of the reservoir storage in period t, m^3
Q_t, $Q_{\max}$	The generation discharge and its upper limit in period t, m^3/s
N_t, $N_{\max}$	The electric output in period t and the installed capacity, kW.h
Z_t, $Z_{dr,t}$	The ending and tail water levels of the reservoir in period t, m
$f(*)$	The function between reservoir water level and its water storage
$g(*)$	The function between tail water level and its generation discharge

2.1.2. Recursive equation considering the expected future value

In the classical SDP, the optimal reservoir decision can be derived by solving the following recursive equation (Tejada-Guibert et al., 2010):

$$f_t(H_t) = \max_{s_t}\{E_{q_t|H_t}B_t(s_{t-1}, q_t, s_t) + E_{q_t|H_t} \cdot E_{H_{t+1}|q_t,H_t} \cdot f_{t+1}(H_{t+1}) \tag{8}$$

where $E_{q_t|H_t}$ is the conditional expectation operator for the inflow q_t in period t for a specific state H_t; and $E_{H_{t+1}|q_t,H_t}$ is the conditional expectation operator for the state in the next stage, which is depended on the state and decision in period t.

In this study, the inflow and the beginning storage of period t (q_t and s_{t-1}) are taken as the state variables, and the ending storage of period t (s_t) is taken as the decision variable. Therefore, the recursive equation is defined as: