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Research papers

Application of the MacCormack scheme to overland flow routing for high-spatial resolution distributed hydrological model

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ABSTRACT

Although process-based distributed hydrological models (PDHMs) are evolving rapidly over the last few decades, their extensive applications are still challenged by the computational expenses. This study attempted, for the first time, to apply the numerically efficient MacCormack algorithm to overland flow routing in a representative high-spatial resolution PDHM, i.e., the distributed hydrology-soil-vegetation model (DHSVM), in order to improve its computational efficiency. The analytical verification indicates that both the semi and full versions of the MacCormack schemes exhibit robust numerical stability and are more computationally efficient than the conventional explicit linear scheme. The full-version outperforms the semi-version in terms of simulation accuracy when a same time step is adopted. The semi-MacCormack scheme was implemented into DHSVM (version 3.1.2) to solve the kinematic wave equations for overland flow routing. The performance and practicality of the enhanced DHSVM-MacCormack model was assessed by performing two groups of modeling experiments in the Mercer Creek watershed, a small urban catchment near Bellevue, Washington. The experiments show that DHSVM-MacCormack can considerably improve the computational efficiency without compromising the simulation accuracy of the original DHSVM model. More specifically, with the same computational environment and model settings, the computational time required by DHSVM-MacCormack can be reduced to several dozen minutes for a simulation period of three months (in contrast with one day and a half by the original DHSVM model) without noticeable sacrifice of the accuracy. The MacCormack scheme proves to be applicable to overland flow routing in DHSVM, which implies that it can be coupled into other PHDMs for watershed routing to either significantly improve their computational efficiency or to make the kinematic wave routing for high resolution modeling computational feasible.

1. Introduction

Overland flow is one of the major components of the hydrological cycle and has the most intimate interactions with human beings because of their coexistence in space and time (Wong, 2011). It is normally unsteady and non-uniform and, therefore, can be described by the St. Venant equations. Owing to the highly nonlinear nature of these equations which involve a high degree of complexity in their computation, various approximations of the St. Venant equations have been proposed for solving overland flow problems. The kinematic wave (KW) model, which was first developed by Lighthill and Whitham (1955), is one of such approximations and proves to be adequate for most practical overland flow situations (Akan and Houghtalen, 2003). The major assumption of the KW model is that the acceleration and pressure terms in the momentum equation are insignificant and, consequently, the friction slope is equal to the terrain slope (Miller, 1984). The KW model is essentially a set of nonlinear hyperbolic partial differential equations for which analytical solutions cannot be obtained except for a few idealized conditions. The finite difference (FD) numerical methods, which are generally classified into explicit and implicit schemes, are therefore frequently used to solve the KW equations. Both the explicit and implicit FD methods have comparative strengths and weaknesses. The explicit FD schemes are easy to formulate and program, but are subjected to a

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necessary and insufficient condition for stability known as the Courant-Friedrichs-Lewy stability (CFL) condition (Chow et al., 1988). The CFL condition imposes a restriction on the workable time steps, which limits the computational efficiency and the practical applications of the explicit FD method. The implicit FD schemes, on the other hand, are unconditionally stable, but suffer from (i) high computational complexity due to the matrix operations; (ii) large memory demand; and (iii) deficiency in their applications to nonlinear problems (Kazezyılmaz-Alhan et al., 2005; Huang and Lee, 2013).

Process-based distributed hydrological models (PDHMs) are evolving rapidly over the last few decades (Paniconi and Putti, 2015). This is partly spurred by the tremendous advances in computing power, programming techniques and data availability; and partly by the increasing demands for spatially distributed hydrological simulations, impact assessments and interdisciplinary studies (Beven, 2011; Fenicia et al., 2016). Nevertheless, the extensive applications of PDHMs are still challenged by their computational burden since PDHMs are mostly associated with solving nonlinear partial differential equations over large domains at fine spatiotemporal resolutions (Fatichi et al., 2016). In PDHMs such as the distributed hydrology-soil-vegetation model (DHSVM) (Wigmosta et al., 1994), the geomorphology-based hydrological model (GBHM) (Yang, 1998), and the water flow and balance simulation model (WaSiM) (Schulla and Jasper, 2007), the routing of overland flow is usually described by the KW equations, owing to its simplicity and satisfactory accuracy compared to the St. Venant equations (Jain and Singh, 2005; Tsai and Yang, 2005; Yu and Duan, 2014). However, the computational efficiency of these models would be strictly constrained in case of using the conventional explicit FD schemes to solve the KW equations, because it usually requires very small time increments to comply with the CFL condition. For example, DHSVM routes the KW overland flow with the explicit linear scheme. We have tested it using the model's test site data, which corresponds to a small urban watershed (31 km^2) at a spatial resolution of 30 m, and found that it needs almost 186 h to complete a 2.25 years simulation. The test was carried out on a Lenovo notebook PC with an Intel Core i7-2620 M CPU and 8 GB of RAM.

Considering the complexity of the real world, and the strong spatial heterogeneity of land surface characteristics, more attention is increasingly paid to high-spatial resolution PDHMs for a refined representation of hydrological processes (Ochoa-Rodriguez et al., 2015). The computational efficiency of PDHMs is even worse when they are applied to a large study domain with a high spatial resolution, since smaller spatial steps require much smaller time steps to achieve a stable solution to the KW equations with an explicit FD scheme. Thus, it is of great significance and practical importance to propose more efficient numerical methods to solve the KW equations for overland flow routing to reduce the computational cost.

The MacCormack FD method, which was initially proposed to solve the time-dependent compressive Navier-Stokes equations, is a popular and widely-used numerical algorithm in computational hydraulics (MacCormack, 2003; Tseng, 2010). Recently, some researchers have successfully applied the MacCormack algorithm to KW overland flow problems. Kazezyılmaz-Alhan et al. (2005) investigated the reliability of the explicit MacCormack scheme and compared it to the available analytical solutions and to a 4-point implicit FD method. They concluded that the MacCormack algorithm is computationally more efficient than the 4-point implicit method, although they are comparable in accuracy. Tseng (2010) applied the unconditionally stable implicit MacCormack scheme to solve the KW problem and demonstrated that it is a simple, accurate, highly stable, and efficient solver. Huang and Lee (2013) reformulated the implicit MacCormack scheme and then applied it to two mountainous watersheds for 2-dimensional (2D) runoff simulations. They reported that the proposed method was significantly superior to the conventional algorithm in terms of computational efficiency.

These previous studies consistently revealed the clear advantage of the MacCormack scheme over the other conventional numerical methods for solving the KW equations. None of them, however, have tested the applicability and advantage of the MacCormack scheme with a PDHM. This study, therefore, applied the MacCormack scheme to the KW overland flow routing in a representative high-spatial resolution PDHM (i.e., the DHSVM model). More specifically, the behaviors of the semi and full versions of the MacCormack schemes were evaluated against analytical solutions for two synthetic overland flow cases with uniform rainfalls. Then the semi-MacCormack algorithm was implemented into DHSVM to improve the efficiency of routing its overland flow. The performance and practicability of the enhanced DHSVM model (DHSVM-MacCormack) were examined by carrying out two groups of modeling experiments in a small urban watershed in Washington.

2. Methods

The method section is organized as follows. Sections 2.1 and 2.2 briefly introduce the KW model and the MacCormack numerical scheme, respectively. Section 2.3 describes the approach of implementing the MacCormack scheme into DHSVM. Finally, in Section 2.4, the methods of evaluating the performances of the MacCormack schemes are presented.

2.1. Kinematic wave

The 1-dimensional (1D) KW model for overland flow routing is defined by the following continuity and momentum equations.

Continuity Equation:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_e \tag{1}$$

where Q is the discharge, A is the cross-sectional area of flow, x is the downslope distance, t is the time, and q_e is the rainfall excess rate per unit flow length.

Momentum Equation:

$$S_0 = S_f \tag{2}$$

in which S_o is the bed slope and S_f is the friction slope. The momentum equation can be expressed equivalently to the following relationship between Q and A.

$$A = \alpha Q^{\beta} \tag{3}$$

By combining Eq. (2) with the Manning's equation, Eq. (3) can be derived as follows:

$$A = \left(\frac{nP^{2/3}}{C_n \sqrt{S_f}} \right) Q^{3/5} \tag{4}$$

where n is the Manning's roughness coefficient; P is the wetted perimeter, which can be considered equal to flow width for shallow water flow; and C_n equals to 1 for metric units and 1.49 for English units. Thus, $\alpha = \left(nP^{2/3}/(C_n\sqrt{S_o})\right)^{3/5}$ and $\beta = 3/5$.

Differentiation of Eq. (3) with respect to t and substitution of this into Eq. (1) gives:

$$\frac{\partial Q}{\partial x} + \alpha\beta Q^{\beta-1}\frac{\partial Q}{\partial t} = q_e \tag{5}$$