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An approximate analytical solution for describing surface runoff and sediment transport over hillslope

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ABSTRACT

Soil and water loss from farmland causes land degradation and water pollution, thus continued efforts are needed to establish mathematical model for quantitative analysis of relevant processes and mechanisms. In this study, an approximate analytical solution has been developed for overland flow model and sediment transport model, offering a simple and effective means to predict overland flow and erosion under natural rainfall conditions. In the overland flow model, the flow regime was considered to be transitional with the value of parameter β (in the kinematic wave model) approximately two. The change rate of unit discharge with distance was assumed to be constant and equal to the runoff rate at the outlet of the plane. The excess rainfall was considered to be constant under uniform rainfall conditions. The overland flow model developed can be further applied to natural rainfall conditions by treating excess rainfall intensity as constant over a small time interval. For the sediment model, the recommended values of the runoff erosion calibration constant (c_r) and the splash erosion calibration constant (c_f) have been given in this study so that it is easier to use the model. These recommended values are 0.15 and 0.12, respectively. Comparisons with observed results were carried out to validate the proposed analytical solution. The results showed that the approximate analytical solution developed in this paper closely matches the observed data, thus providing an alternative method of predicting runoff generation and sediment yield, and offering a more convenient method of analyzing the quantitative relationships between variables. Furthermore, the model developed in this study can be used as a theoretical basis for developing runoff and erosion control methods.

1. Introduction

Soil erosion and nutrient loss caused by rainfall contribute significantly to land degradation and nonpoint source pollution. The processes of runoff generation and soil erosion need to be better understood and quantified. Predicting rainfall runoff is critical in predicting soil erosion (Raff and Ramirez, 2005). Mathematical models related to runoff generation and sediment transport have been used as an effective way of predicting soil and water loss during rainfall, with the process of overland flow generation usually described using Saint-Venant equations (i.e., the continuity equation and the momentum equation) (Wang et al., 2002). However, it is very difficult to obtain analytical solutions as these equations are highly nonlinear (Wang et al., 2006), meaning that only numerical techniques can be used (Ying et al., 2004; Crossley et al., 2003; Lackey and Sotiropoulos, 2005). When the acceleration term is ignored, the Saint-Venant equations can be simplified using diffusion wave equations. Hayami (1951) developed an analytical solution for diffusion wave equations (in rivers) by using a disturbance function; Kazeyilmaz-Ahan and Medina (2007) and Kazeyilmaz-Ahan (2012) provided a solution for overland flow, then improved the solution for diffusion waves applied to overland flow by employing the De Hoog algorithm. Govindaraju et al. (1988) and Rao and Kavvas (1991) suggested that diffusion wave equations may be better suited to steep rough slopes. When the acceleration and pressure terms are ignored, the Saint-Venant equations can be expressed as kinematic wave equations. The explicit analytical solution (for uniform rainfall in time and space) to kinematic wave equations was first given by Henderson and Wooding (1964). Parlange et al. (1981) developed a general analytical solution to the kinematic flow for variable rainfall. Sander et al. (1990) obtained the solution for when infiltration rate exceeds rainfall

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intensity by considering the excess rainfall to be equal to the hydraulic conductivity. Luce and Cundy (1992) modified the kinematic wave equation by using the Philips infiltration equation to predict excess rainfall, and applied the solution to field data. Tsai and Yang (2005) used the method of characteristics integrated with cubic spline interpolation to compute one-dimensional and two-dimensional kinematic wave models. To obtain the closed form of the analytical solution, Mizumura (2006) assumed that the unit discharge in Manning's formula was a parabolic curve. Gottardi and Venutelli (2008) suggested an accurate time integration method for diffusion wave and kinematic wave equations. Morooka et al. (2016) proposed a new theoretical framework (theory of stochastic process) to estimate rainfall runoff.

Foster (1986) defined detachment as the soil particles being removed from the soil surface and transport as the detached soil particles moving to some location away from the detachment point. The processes of detachment and transport can be described using the continuity equation, with sediment discharge considered to be a function of water flow, soil properties, and topography (Bennet, 1974). Most of the physical model developed in this study was based on the continuity equation. The ANSWERS (Areal Nonpoint Source Watershed Response Simulation) model was used to predict erosion of agricultural watersheds by treating runoff and erosion as independent processes (Beasley et al., 1980). The WEEP (Water Erosion Prediction Project) model (Nearing et al., 1989) divides rainfall erosion into rill and interrill areas, and calculates the erosion in these areas separately. EUROSEM (Euro Open Soil Erosion Model) (Morgan et al., 1998) is an event-based model which assumes that a few events every year are the main contributors to erosion; it also calculates rill erosion and interrill erosion separately.

For this paper, an approximate analytical model for predicting overland flow and rainfall erosion was developed. The overland flow model was then used to predict flow and erosion under natural rainfall conditions. The recommended values for the runoff erosion calibration and splash erosion calibration constants are defined in this paper, producing a simple and effective model for predicting overland flow and erosion under natural rainfall conditions.

2. Theoretical analysis

2.1. Overland flow

The kinematic wave model was used to describe the process of overland flow generation:

$$\frac{\partial Q(x,t)}{\partial x} + \frac{\partial h(x,t)}{\partial t} = r_e(t) \tag{1}$$

$$Q(x,t) = \alpha h(x,t)^{\beta} \tag{2}$$

where $Q(x, t)$ is the unit discharge (cm^2/min), $h(x, t)$ is the depth of overland flow (cm), x is the distance along the overland plane (cm), t is rainfall duration (min), r_e is excess rainfall (cm/min), $\alpha = J^{1/2}/n$ ($cm^{1/3}$/min), J is the overland slope (cm/cm), n is Manning's roughness coefficient (min/$cm^{1/3}$), and β is an exponent that reflects the degree of turbulence ($5/3 < \beta < 3$) (Emmett, 1970; Sander et al., 2014).

Assuming that the change rate of unit discharge with distance is constant, and also equals the discharge per unit area at the outlet of the plane (Moore, 1985; Agnese et al., 2001), the unit discharge can be expressed as:

$$Q(x,t) = q(t)x \tag{3}$$

where $q(t)$ is the discharge per unit area (cm/min).

Combining Eqs. (2) and (3) yields the depth of the overland flow:

$$h(x,t) = \left(\frac{x}{\alpha}q(t)\right)^{\frac{1}{\beta}} \tag{4}$$

Differentiating Eq. (4) yields the change rate of flow depth with time:

$$\frac{\partial h(x,t)}{\partial t} = \frac{1}{\beta}\left(\frac{x}{\alpha}\right)^{\frac{1}{\beta}} q(t)^{\frac{1-\beta}{\beta}} \frac{dq(t)}{dt} \tag{5}$$

Substituting Eq. (5) into Eq. (1) gives:

$$q(t) + \frac{1}{\beta}\left(\frac{x}{\alpha}\right)^{\frac{1}{\beta}} q(t)^{\frac{1-\beta}{\beta}} \frac{dq(t)}{dt} = r_e(t) \tag{6}$$

Integrating both sides of Eq. (6) with respect to distance x (from 0 to L) gives:

$$\int_0^L r_e(t)dx - \int_0^L q(t)dx = \frac{1}{\beta} q(t)^{\frac{1-\beta}{\beta}} \frac{dq(t)}{dt} \int_0^L \left(\frac{x}{\alpha}\right)^{\frac{1}{\beta}} dx \tag{7}$$

The result of the integration is:

$$r_e(t)L - q(t)L = q(t)^{\frac{1-\beta}{\beta}} \frac{dq(t)}{dt} \frac{L}{1+\beta} \left(\frac{L}{\alpha}\right)^{1/\beta} \tag{8}$$

Separating variables $q(t)$ and t, we obtain:

$$dt = \frac{q(t)^{\frac{1-\beta}{\beta}}}{r_e(t) - q(t)} \frac{1}{\beta+1} \left(\frac{L}{\alpha}\right)^{1/\beta} dq(t) \tag{9}$$

Integrating Eq. (9):

$$t - t_p = \frac{\left(\frac{L}{\alpha}\right)^{\frac{1}{\beta}}}{(\beta+1)} \int \frac{q(t)^{\frac{1-\beta}{\beta}}}{r_e(t) - q(t)} dq(t) \tag{10}$$

where t_p is the time of ponding (min).

2.1.1. The approximate solution under uniform rainfall conditions

Most of the rainfall experiments were carried out under uniform rainfall conditions. In order to obtain an approximate solution under uniform rainfall conditions, it is necessary to estimate accurately the parameters, particularly β, which reflects the degree of turbulence and the type of flow regime that the overland flow belongs to. In this research, the flow regime is treated as transitional, with the value of β approximated to be two (Horton, 1938; Agnese et al., 2001; Singh, 2002). In addition, the excess rainfall under uniform rainfall intensity conditions was considered to be a constant (i.e., rainfall intensity minus stable infiltration rate) when obtaining the solution to the integration of Eq. (10) (Agnese et al., 2001; Moore, 1985). Then, Eq. (10) can be approximated as:

$$t - t_p = \frac{1}{3}\left(\frac{L}{\alpha}\right)^{\frac{1}{2}} \int_{q(t_p)}^{q(t)} \frac{q(t)^{-\frac{1}{2}}}{r_e - q(t)} dq(t) \tag{11}$$

Hence, for uniform rainfall intensity, the outflow rate in the rising stage can be expressed as:

$$q(t) = \tanh^2\left(\frac{3}{2}\sqrt{\frac{\alpha r_e}{L}}(t - t_p)\right) r_e \tag{12}$$

The unit discharge and flow depth in the rising stage can be expressed as:

$$Q(x,t) = \tanh^2\left(\frac{3}{2}\sqrt{\frac{\alpha r_e}{L}}(t - t_p)\right) r_e x \tag{13}$$

$$h(x,t) = \sqrt{\frac{r_e x}{\alpha}} \tanh\left(\frac{3}{2}\sqrt{\frac{\alpha r_e}{L}}(t - t_p)\right) \tag{14}$$