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Alexandru Chirvasitu, Souleiman Omar Hoche, Paweł Kasprzak

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FUNDAMENTAL ISOMORPHISM THEOREMS FOR QUANTUM GROUPS

ALEXANDRU CHIRVASITU, SOULEIMAN OMAR HOCHE, PAWEŁ KASPRZAK

Abstract. The lattice of subgroups of a group is the subject of numerous results revolving around the central theme of decomposing the group into "chunks" (subquotients) that can then be compared to one another in various ways. Examples of results in this class would be the Noether isomorphism theorems, Zassenhaus' butterfly lemma, the Schreier refinement theorem, the Dedekind modularity law, and last but not least the Jordan-Hölder theorem.

We discuss analogues of the above-mentioned results in the context of locally compact quantum groups and linearly reductive quantum groups. The nature of the two cases is different: the former is operator algebraic and the latter Hopf algebraic, hence the corresponding two-part organization of our study. Our intention is that the analytic portion be accessible to the algebraist and vice versa.

The upshot is that in the locally compact case one often needs further assumptions (integrability, compactness, discreteness). In the linearly reductive case on the other hand, the quantum versions of the results hold without further assumptions. Moreover the case of compact / discrete quantum groups is usually covered by both the linearly reductive and the locally compact framework, thus providing a bridge between the two.

Key words: locally compact quantum group, discrete quantum group, linearly reductive quantum group, Zassenhaus lemma, Schreier refinement theorem, Jordan-Hölder theorem

MSC 2010: 46L89; 46L85; 46L52; 16T20; 20G42

Introduction

The theory of quantum groups has been a rich and fruitful one, as evidenced by the many excellent monographs on the subject [5, 20, 21] and the references therein. As the vastness of the field would by necessity make any attempt at documenting the literature incomplete, we cite only a select few sources in this introduction and instead refer the reader to the papers that are more immediately relevant for us in the main body of the paper.

The appropriately ill-defined concept of a quantum group is flexible enough to allow for several branches of the theory, that continue to develop vigorously but largely independently. In this paper we draw a rough distinction between two flavors of quantum-group-theoretic results: those of an analytic nature, where the objects to be studied are operator algebras (C^* or von Neumann) that mimic the behavior of algebras of (continuous, essentially bounded, etc.) functions on a locally compact group (see e.g. [23] and the references cited there for this perspective), and those of a purely algebraic character, whereby the quantum groups are recast as Hopf algebras ([12] itself, where the term 'quantum group' seems to have been coined, or numerous other sources, such as [2], where a category of quantum groups is defined explicitly).

While there is a common core of notions to the two branches (irreducible representations, Pontryagin-type duality, etc.), the techniques used in practice and the attendant technical