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Pure gaps on curves with many rational places

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We consider the algebraic curve defined by $y^m = f(x)$ where $m \geq 2$ and $f(x)$ is a rational function over $\mathbb{F}_q$. We extend the concept of pure gap to **c**-gap and obtain a criterion to decide when an s-tuple is a **c**-gap at s rational places on the curve. As an application, we obtain many families of pure gaps at two rational places on curves with many rational places.

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1. Introduction

Since Goppa introduced the Algebraic Geometric codes in the eighties [10], a lot of effort has been directed towards obtaining examples of codes with good parameters through different types of algebraic curves. It is well known that codes with optimal parameters are expected from algebraic curves with many rational places over finite fields. These are curves whose numbers of rational places are equal or close to the Hasse–Weil upper bound or other known bounds that may be specific to the curve. For a projective, absolutely irreducible, non-singular algebraic curve $\mathcal{X}$ of genus g over $\mathbb{F}_q$, the Hasse–Weil upper bound on the number of $\mathbb{F}_q$-rational places is $q+1+2g\sqrt{q}$. When this quantity is attained, the curve $\mathcal{X}$ is said to be *maximal*. Maximal curves only exist over $\mathbb{F}_{q^2}$.

In [6] and [7] Garcia, Kim and Lax exploit a local property at a rational place on an algebraic curve in order to improve the minimum distance of the code. Specifically, they show that the existence of ℓ consecutive gaps at a rational place can be used to increase the classical upper bound for the minimum distance by ℓ units. In [15] Homma and Kim obtain similar results using gaps at two rational places. They also define a special type of gap that they call *pure gap* and obtain further improvements. Then they use pure gaps to refine the parameters of codes constructed from the Hermitian curve over $\mathbb{F}_{q^2}$. In [3] Carvalho and Torres consider in detail gaps and pure gaps at more than two places. Since then, pure gaps have been exhaustively studied to show that they provide many other good codes; see [2,4,18,21,23,26,27]. The scope of these papers varies depending on the algebraic curve and the number of places considered.

The majority of maximal curves and curves with many rational places has a plane model of Kummer-type. For those curves with affine equation given by

$$y^m = f(x)^\lambda \text{ where } m \geq 2, \lambda \geq 1 \text{ and } f(x) \text{ is a separable polynomial over } \mathbb{F}_q,$$

general results on gaps and pure gaps can be found in [1,4,16,26]. Applications to codes on particular curves such as the Giulietti–Korchmáros curve, the Garcia–Güneri–Stichtenoth curve, and quotients of the Hermitian curve can be found in [17,25,28]. In [1] the authors use a decomposition of certain Riemann–Roch vector spaces due to Maharaj (Theorem 2.2) to describe gaps at one place arithmetically. The same idea has been used to investigate gaps and pure gaps at several places [2,4,16,17,26–28]. Here we continue exploring its capabilities by considering a different setting.

In this work we consider a Kummer-type curve defined by

$$y^m = f(x) \text{ where } m \geq 2 \text{ and } f(x) \text{ is a rational function over } \mathbb{F}_q.$$

We extend the concept of pure gap to $\mathbf{c}$-*gap* and provide an arithmetical criterion to decide when an s-tuple is a $\mathbf{c}$-gap at s rational places. This result is then heavily used to obtain many families of pure gaps at two places on the curves obtained by Giulietti and Korchmáros in [9], Garcia and Quoos in [8], and Garcia, Güneri and Stichtenoth in [5]. All these curves are known to have many rational places.