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On the 3-extendability of quaternary linear codes

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ABSTRACT

We consider the extendability of linear codes over $\mathbb{F}_4$, the field of order four. Let $\mathcal{C}$ be $[n, k, d]_4$ code with $d \equiv 1 \pmod 4$, $k \geq 3$. The weight spectrum modulo 4 (4-WS) of $\mathcal{C}$ is defined as the ordered 4-tuple (w_0, w_1, w_2, w_3) with $w_0 = \frac{1}{3}\sum_{4|i>0} A_i$, $w_j = \frac{1}{3}\sum_{i\equiv j \pmod 4} A_i$ for $j = 1, 2, 3$. We prove that $\mathcal{C}$ is 3-extendable if $w_0 + w_2 = \theta_{k-2}$ and if either (a) $w_1 - w_0 < 4^{k-2} + 4 - \theta_{k-3}$; (b) $w_1 - w_0 > 10 \cdot 4^{k-3} - \theta_{k-3}$ or (c) $(w_0, w_1) = (\theta_{k-3}, 6 \cdot 4^{k-3})$. We also give a sufficient condition for the l-extendability of $[n, k, d]_4$ codes with $d \equiv 4 - l \pmod 4$, $k \geq 3$ for $l = 1, 2, 3$ when $w_0 + w_2 = \theta_{k-2} + 2 \cdot 4^{k-2}$.

1. Introduction

Let $\mathbb{F}_q$ denote the field of q elements. We denote by $\mathbb{F}_q^n$ the set of n-tuples over $\mathbb{F}_q$. The *weight* of a vector $c \in \mathbb{F}_q^n$, denoted by $wt(c)$, is the number of nonzero entries in c. An $[n, k, d]_q$ *code* or a *q-ary linear code* of length n with dimension k and minimum weight d is a k-dimensional subspace of $\mathbb{F}_q^n$ whose minimum weight of nonzero codewords is d. The weight distribution of $\mathcal{C}$ is the list of numbers A_i which is the number of codewords

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of $\mathcal{C}$ with weight i. The weight distribution with $(A_0, A_d, ...) = (1, \alpha, ...)$ is also expressed as $0^1 d^\alpha \cdots$. For an $[n, k, d]_q$ code $\mathcal{C}$ with generator matrix G, $\mathcal{C}$ is called *l-extendable* if there exist l vectors $h_1, \dots, h_l \in \mathbb{F}_q^k$ such that the extended matrix $[G\ h_1^{\mathrm{T}}\ \cdots\ h_l^{\mathrm{T}}]$ generates an $[n+l, k, d+l]_q$ code $\mathcal{C}'$, and $\mathcal{C}'$ is an *l-extension* of $\mathcal{C}$. Especially when $l = 1$, $\mathcal{C}$ is simply called *extendable* and $\mathcal{C}'$ is an *extension* of $\mathcal{C}$. In this paper, we deal with the extendability of quaternary linear codes. Extension theorems are employed to find optimal linear codes to construct new codes from old ones or to prove the nonexistence of codes with certain parameters; see [8,15] for ternary linear codes and [2,11] for linear codes over $\mathbb{F}_q$. The l-extendability of $[n, k, d]_4$ codes was investigated in [9,12] for $l = 1$ when d is odd and in [5,13,14,16] for other cases.

Let $\mathcal{C}$ be an $[n, k, d]_q$ code with $d \not\equiv 0 \pmod{q}$. We define the *weight spectrum modulo* q (q-**WS**) as the q-tuple $(w_0, w_1, \dots, w_{q-1})$ with

$$w_0 = \frac{1}{q-1} \sum_{q|i>0} A_i, \quad w_j = \frac{1}{q-1} \sum_{i \equiv j \pmod{q}} A_i \text{ for } j = 1, 2, \dots, q-1.$$

From now on in this section, let $q = 4$. Denote by θ_j the number of points in $\mathrm{PG}(j, 4)$, i.e., $\theta_j = (4^{j+1} - 1)/3$. We set $\theta_0 = 1$ and $\theta_j = 0$ for $j < 0$ for convenience.

As for the known extension theorems for linear codes over $\mathbb{F}_4$, see [5,14] for the case when $d \equiv 2 \pmod 4$ and [5,7,9,12] for the case when $d \equiv 3 \pmod 4$. In this paper, we mainly consider the case when $d \equiv 1 \pmod 4$. The following result is already known for such a case.

Theorem 1.1 *([5]). Let $\mathcal{C}$ be an $[n, k, d]_4$ code with 4-WS $(w_0, ..., w_3)$, $k \geq 3$, $d \equiv 1 \pmod 4$. Then $\mathcal{C}$ is 3-extendable if one of the following conditions holds:*

(a) $w_0 = \theta_{k-4}$,
(b) $w_0 = \theta_{k-3}$ *and* $w_2 = 3 \cdot 4^{k-2}$,
(c) $w_j = 0$ *for* $j = 2$ *or* 3.

The aim of this paper is to give some new sufficient conditions for the 3-extendability of $[n, k, d]_4$ codes with $d \equiv 1 \pmod 4$. We consider the cases $w_0 + w_2 = \theta_{k-2}$ or $\theta_{k-2} + 2 \cdot 4^{k-2}$. The following four theorems are our main results.

Theorem 1.2. *Let $\mathcal{C}$ be an $[n, k, d]_4$ code with 4-WS $(w_0, ..., w_3)$ with $w_0 + w_2 = \theta_{k-2}$, $k \geq 3$, $d \equiv 1 \pmod 4$. Then $\mathcal{C}$ is 3-extendable if either*

(a) $w_1 - w_0 < 4^{k-2} + 4 - \theta_{k-3}$ *or*
(b) $w_1 - w_0 > 10 \cdot 4^{k-3} - \theta_{k-3}$.

Whilst one can not apply Theorem 1.2 when $w_1 - w_0 = 6 \cdot 4^{k-3} - \theta_{k-3}$, we prove the following.