

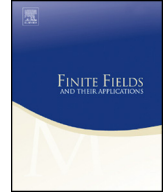


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# Algebraic geometric codes on many points from Kummer extensions

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## ABSTRACT

For Kummer extensions defined by  $y^m = f(x)$ , where  $f(x)$  is a separable polynomial over the finite field  $\mathbb{F}_q$ , we compute the number of Weierstrass gaps at two totally ramified places. For many totally ramified places we give a criterion to find pure gaps at these points and present families of pure gaps. We then apply our results to construct  $n$ -points algebraic geometric codes with good parameters.

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**1. Introduction**

In the early eighties tools from algebraic geometry were applied by V. Goppa to construct linear codes using algebraic curves over finite fields, see [8]. Nowadays these codes are called algebraic–geometric codes, AG codes for short. The starting point in the construction of an AG code is a projective, absolutely irreducible, non singular algebraic curve  $\mathcal{X}$  of genus  $g \geq 1$  defined over the finite field  $\mathbb{F}_q$  with cardinality  $q$ . Let  $F = \mathbb{F}_q(\mathcal{X})$  be its function field with  $\mathbb{F}_q$  being the field of constants. Consider  $Q_1, \dots, Q_n$  pairwise distinct rational places on  $F$ . Let  $D = Q_1 + \dots + Q_n$  and  $G$  be divisors such that  $Q_i$  is not in the support of  $G$  for  $i = 1, \dots, n$ . The linear code  $C_\Omega(D, G)$  is defined by

$$C_\Omega(D, G) = \{(\text{res}_{Q_1}(\eta), \dots, \text{res}_{Q_n}(\eta)) \mid \eta \in \Omega(G - D)\} \subseteq \mathbb{F}_q^n,$$

where  $\Omega(G - D)$  is the space of  $\mathbb{F}_q$ -rational differentials  $\eta$  on  $\mathcal{X}$  such that either  $\eta = 0$  or  $\text{div}(\eta) \succeq G - D$  and  $\text{res}_{Q_j} \eta$  is the residue of  $\eta$  at  $Q_j$ .

The code  $C_\Omega(D, G)$  has length  $n$  and dimension  $k = i(G - D) - i(G)$  where  $i(G)$  denotes the speciality index of the divisor  $G$ . We say that  $C_\Omega(D, G)$  is an  $[n, k, d]$ -code where  $d$  denotes the minimum distance of the code. One of the main features of this code is that its minimum distance  $d$  satisfies the classical Goppa bound, namely

$$d \geq \deg G - (2g - 2).$$

The integer  $d^* = \deg G - (2g - 2)$  is usually called the *designed minimum distance*. One way to obtain codes with good parameters is to find codes that improve the designed minimum distance.

If  $G = \alpha P$  for some rational place  $P$  on  $F$  and  $D$  is the sum of other rational places on  $\mathcal{X}$ , then the code  $C_\Omega(D, G)$  is called an *one-point AG code*. Analogously, if  $G = \alpha_1 P_1 + \dots + \alpha_n P_n$  for  $n$  distinct rational places  $P_1, \dots, P_n$  on  $\mathcal{X}$ , then  $C_\Omega(D, G)$  is called a *n-point AG code*. For a more detailed introduction to AG codes, see [13,21].

For a one-point divisor  $G = \alpha P$  on the function field  $F$ , Garcia, Kim, and Lax [6,7] improved the designed minimum distance using the arithmetical structure of the Weierstrass semigroup at the rational place  $P$ . For a two-point divisor  $G = \alpha_1 P_1 + \alpha_2 P_2$ , Homma and Kim [11] introduced the notion of pure gaps and obtained similar results. By choosing  $\alpha_1$  and  $\alpha_2$  satisfying certain arithmetical conditions depending on the structure of the Weierstrass semigroup at  $P_1$  and  $P_2$ , they improved the designed minimum distance. Matthews [17] showed that for an arbitrary curve there exist two-point AG codes that have better parameters than any comparable one-point AG code constructed from the same curve. Finally, for divisors  $G = \alpha_1 P_1 + \dots + \alpha_n P_n$  at  $n$  distinct rational places on  $\mathcal{X}$ , results from the theory of generalized Weierstrass semigroups and pure gaps were obtained by Carvalho and Torres [2]. They have been used to obtain AG codes whose minimum distance beats the classical Goppa bound on the minimum distance, see Theorem 2.4.

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