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Generalized bilinear forms graphs and MRD codes over a residue class ring ☆

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We investigate the generalized bilinear forms graph Γ_d over a residue class ring $\mathbb{Z}_{p^s}$. The graph Γ_d is a connected vertex-transitive graph, and we completely determine its independence number, clique number, chromatic number and maximum cliques. We also prove that cores of both Γ_d and its complement are maximum cliques. The graph Γ_d is useful for error-correcting codes. We show that there is a largest independent set of Γ_d which is a linear MRD code over $\mathbb{Z}_{p^s}$.

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1. Introduction

Throughout, all graphs are *simple* [14] and finite. Let $V(G)$ denote the vertex set of a graph G. For two vertices $x, y \in V(G)$, we write $x \sim y$ if x and y are adjacent. Let $\mathbb{F}_q$ be the finite field with q elements where q is a power of a prime. The cardinality of a set X is denoted by $|X|$.

Let R be a commutative local ring and R^* the set of all units of R. For a subset S of R, let $S^{m\times n}$ be the set of all $m \times n$ matrices over S, and let $S^n = S^{1\times n}$. Let $GL_n(R)$ be the set of $n \times n$ invertible matrices over R. Write ${}^tA$ as the transpose matrix of a matrix A. Denote by I_r (I for short) the $r \times r$ identity matrix, and $\mathrm{diag}(A_1, \ldots, A_k)$ a block diagonal matrix where A_i is an $m_i \times n_i$ matrix. Let $0_{m,n}$ (0 for short) be the $m \times n$ zero matrix and $0_n = 0_{n,n}$.

For $0 \neq A \in R^{m\times n}$, by Cohn's definition [6], the *inner rank* of A, denoted by $\rho(A)$, is the least positive integer r such that $A = BC$ where $B \in R^{m\times r}$ and $C \in R^{r\times n}$. Let $\rho(0) = 0$. For $A \in R^{m\times n}$, it is clear that $\rho(A) \leq \min\{m, n\}$, and $\rho(A) = 0$ if and only if $A = 0$. When R is a field, we have $\rho(A) = \mathrm{rank}(A)$, where $\mathrm{rank}(A)$ is the usual rank of a matrix over a field. For matrices over R, we have (cf. [6,5, Section 5.4]): $\rho(A) = \rho(PAQ)$ where P and Q are invertible matrices over R; $\rho(A+B) \leq \rho(A)+\rho(B)$; $\rho(AC) \leq \min\{\rho(A), \rho(C)\}$ and $\rho(A_{11}) \leq \rho\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$.

For $A, B \in R^{m\times n}$, the *rank distance* between A and B is defined by

$$\mathrm{d_R}(A, B) = \rho(A - B). \tag{1.1}$$

We have that $\mathrm{d_R}(A, B) = 0 \Leftrightarrow A = B$, $\mathrm{d_R}(A, B) = \mathrm{d_R}(B, A)$ and $\mathrm{d_R}(A, B) \leq \mathrm{d_R}(A, C)+ \mathrm{d_R}(C, B)$, for all matrices of appropriate sizes A, B, C over R.

Let $\mathbb{Z}_{p^s}$ denote the *residue class ring* of integers modulo p^s, where p is a prime and s is a positive integer. Then $\mathbb{Z}_{p^s}$ is a *Galois ring*, a commutative *local ring*, a finite *principal ideal ring* (cf. [20,27]). The principal ideal (p) is the unique maximal ideal of $\mathbb{Z}_{p^s}$, and denoted by J_{p^s}. Note that J_{p^s} is also the *Jacobson radical* of $\mathbb{Z}_{p^s}$. When $s = 1$, $\mathbb{Z}_p$ is the finite field $\mathbb{F}_p$. We have (cf. [20,27]) that

$$|\mathbb{Z}_{p^s}| = p^s, \quad \left|\mathbb{Z}_{p^s}^*\right| = (p-1)p^{s-1}, \quad |J_{p^s}| = p^{s-1}. \tag{1.2}$$

The residue class ring plays an important role in mathematics and information theory.

The *generalized bilinear forms graph* (*bilinear forms graph* for short) over $\mathbb{F}_q$, denoted by $\Gamma_d(\mathbb{F}_q^{m\times n})$, has the vertex set $\mathbb{F}_q^{m\times n}$ where $m, n \geq 2$, and two distinct vertices A and B are adjacent if $\mathrm{rank}(A - B) < d$ where d is fixed with $2 \leq d \leq \min\{m, n\}$. When $d = 2$, $\Gamma_2(\mathbb{F}_q^{m\times n})$ is the usual *bilinear forms graph* over $\mathbb{F}_q$. The bilinear forms graph has been extensively studied (cf. [3,8,12,16,18,26,28]).

As a natural extension of the generalized bilinear forms graph over $\mathbb{F}_q$, the *generalized bilinear forms graph* (*bilinear forms graph* for short) over $\mathbb{Z}_{p^s}$, denoted by $\Gamma_d(\mathbb{Z}_{p^s}^{m\times n})$ (Γ_d for short), has vertex set $\mathbb{Z}_{p^s}^{m\times n}$ where $m, n \geq 2$, and two distinct vertices A and