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Pure Weierstrass gaps from a quotient of the Hermitian curve

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In this paper, by employing some results on Kummer extensions, we give an arithmetic characterization of pure Weierstrass gaps at many totally ramified places on a quotient of the Hermitian curve, including the well-studied Hermitian curve as a special case. The cardinality of these pure gaps is explicitly investigated. In particular, the numbers of gaps and pure gaps at a pair of distinct places are determined precisely, which can be regarded as an extension of the previous work by Matthews (2001) considered Hermitian curves. Additionally, some concrete examples are provided to illustrate our results.

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1. Introduction

In [7,8] Goppa constructed algebraic geometric codes (AG codes for short) from several rational places by using algebraic curves, which led to an important research line in coding theory. Nowadays a great deal of works are devoted to determining or improving the parameters of AG codes, see [3,5,10,16,17] and the references therein.

Weierstrass semigroups and pure gaps are of significant uses in the construction and analysis of AG codes for their applications in obtaining codes with good parameters (see [4,11]). In [5,6], Garcia, Kim and Lax improved the Goppa bound using the arithmetical structure of the Weierstrass gaps at only one place. Homma and Kim [10] introduced the concept of pure gaps and demonstrated a similar result for a pair of places. And this was generalized to several places by Carvalho and Torres in [3].

Weierstrass semigroups and gaps over specific Kummer extensions were well-studied in the literature. For instance, the authors of [13,14,24] computed the Weierstrass gaps and improved the parameters of one-point AG codes from Hermitian curves. Matthews [19] investigated the Weierstrass semigroup of any collinear places on a Hermitian curve. In [20], Matthews generalized the results of [3,19] by determining the Weierstrass semigroup of arbitrary rational places on the quotient of the Hermitian curve defined by the equation $y^m = x^q + x$ over $\mathbb{F}_{q^2}$ where q is a prime power and $m > 2$ is a divisor of $q + 1$. For general Kummer extensions, the authors in [1,4] recently described the Weierstrass semigroups and gaps at one place and two places. Bartoli, Quoos and Zini [2] gave a criterion to find pure gaps at many places and presented families of pure gaps. In [11,25], Hu and Yang explicitly determined the Weierstrass semigroups and pure gaps at many places, and constructed AG codes with excellent parameters. However, little is known about the numbers of gaps and pure gaps from algebraic curves. Kim [12] obtained lower bounds and upper bounds on the cardinality of gaps at two places, which was pushed forward by Homma [9] to give the exact expression. Applying Homma's theory [9], the authors in [2,18] counted the gaps and pure gaps at two places on Hermitian curves and specific Kummer extensions, respectively.

In this paper, we will study pure Weierstrass gaps of many distinct rational points on the quotient of the Hermitian curve, containing the Hermitian curve as a special case. To be precise, we first focus our attention on characterizing the pure gaps over the Hermitian curve and counting their total number. Then the techniques are employed to determining the number of pure gaps on a quotient of the Hermitian curve. Besides, we explicitly present the numbers of gaps and pure gaps at two distinct rational points, extending the results on Hermitian curves maintained by Matthews [18].

The remainder of the paper is organized as follows. In Section 2 we briefly recall some notations and preliminary results over arbitrary function fields. Section 3 focuses on the pure gaps at many points over Hermitian curves, where the cardinality of pure gap set is determined explicitly through its arithmetic characterization. Finally, in Section 4, we consider the quotient of the Hermitian curves and provide some examples by using our main results.