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The classification of antipodal two-weight linear codes

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By a classical result of Bonisoli, the equidistant linear codes over $\mathrm{GF}(q)$ are, up to monomial equivalence, just the replications of some q-ary simplex code, possibly with added 0-coordinates. We prove an analogous result for the antipodal two-weight linear codes over $\mathrm{GF}(q)$ (that is, one of the two weights is the length of the code): up to monomial equivalence, any such code is ether a replication of a first order generalized Reed–Muller code, or a replication of a 3-dimensional projective code associated with some non-trivial maximal arc in the classical projective plane $\mathrm{PG}(2, 2^s)$, $s \geq 1$.

An equivalent geometric formulation of this result reads as follows: a multiset S of points spanning $\Pi = \mathrm{PG}(m, q)$, $m \geq 2$, for which every hyperplane intersecting S does so in a constant number of points (counted with multiplicity), is either a multiple of a maximal arc in Π (for $m = 2$) or a multiple of the complement of some hyperplane of Π.

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1. Introduction

We assume familiarity with basic facts and notation concerning codes [8], [10] and the classical projective planes [7].

Linear codes with only a few distinct weights are of particular interest in Coding Theory, both for theoretical and practical reasons. The following famous result of Bonisoli [2] characterizes the *equidistant* linear codes (sometimes referred to as *constant weight* linear codes, as they are the linear codes with just one non-zero weight); see also Ward [11] for a short and elegant alternative proof.

Result 1.1 *(Bonisoli's theorem).* Any equidistant linear code over $\mathrm{GF}(q)$ is, up to monomial equivalence, a replication of some q-ary simplex code (that is, some dual q-ary Hamming code), possibly with added 0-coordinates. □

The next class of codes to be considered in this context are the *two-weight* linear codes – linear codes with just two non-zero weights. The investigation of this huge class of codes goes back to Delsarte [6]. It is intimately connected to many other interesting combinatorial objects, such as strongly regular graphs, partial difference sets and certain (multi-)sets of points in finite projective spaces. We refer the reader to the systematic exposition by Calderbank and Kantor [4], which is still the standard reference for this topic.

Even though a classification of all two-weight linear codes is clearly impossible, one can hope to get interesting results for special subclasses. One natural such subclass is formed by the *antipodal* codes of this type, where one of the two weights is the length n of the code.[1] In our recent paper [9], we proved the following characterization for all such codes with the minimal number of codewords with full weight n:

Result 1.2. Let C be any antipodal linear two-weight code over $F = \mathrm{GF}(q)$ and assume that C contains no linearly independent codewords of full weight. Then C is, up to monomial equivalence, a replication of some first order generalized Reed–Muller code over F. □

In Section 2, we provide two preliminary results required for our investigation of antipodal two-weight linear codes in general. First, we prove a slight strengthening of the description of the possible generator matrices given in [3, Corollary 3.4]. Then, using the same approach as in our proof of Result 1.2, we will establish the following important auxiliary result: any antipodal two-weight linear code is a replication of some *projective code* of this type (that is, of a code with dual distance at least 3).

After these preliminaries, we can obtain the desired complete classification of all antipodal two-weight linear codes in Section 3: up to monomial equivalence, the antipodal

1 This class is also of interest in the context of completely regular codes with covering radius $\rho = 2$; see Borges, Rifà and Zinoviev [3, Theorem 3.3].