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Maximum distance separable codes for b-symbol read channels [☆]

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ABSTRACT

Recently, Yaakobi et al. introduced codes for b-symbol read channels, where the read operation is performed as a consecutive sequence of $b > 2$ symbols. In this paper, we establish a Singleton-type bound for b-symbol codes. Codes meeting the Singleton-type bound are called maximum distance separable (MDS) codes, and they are optimal in the sense they attain the maximal minimum b-distance. We introduce a construction method using projective geometry, and then construct several infinite families of linear MDS b-symbol codes over finite fields. The lengths of these codes have a large range. And in some sense, we completely determine the existence of linear MDS b-symbol codes over finite fields for certain parameters.

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1. Introduction

In the traditional information theory, noisy channels are analyzed generally by dividing the message into individual information units. However, with the development of storage technologies, one finds that symbols cannot always be written and read consistently in channels that output overlapping symbols.

In 2011, Cassuto and Blaum [1] first proposed a new coding framework for symbol-pair read channels. The outputs of the read process in the channels are overlapping pairs of symbols. After that, Chee et al. [3] established a Singleton-type bound for symbol-pair codes and considered the constructions of symbol-pair codes meeting the bound. For a complete comprehension of the fruitful results on this topic, please refer to [1–7,9,10] and the references therein.

Recently, Yaakobi et al. [10] generalized the coding framework for symbol-pair read channels to that for b-symbol read channels, where the read operation is performed as a consecutive sequence of $b > 2$ symbols. They also generalized some of the known results for symbol-pair read channels to those for b-symbol read channels.

This paper continues the investigation of codes for b-symbol read channels. We establish a Singleton-type bound for b-symbol codes, and codes meeting this bound are maximum distance separable (MDS). MDS b-symbol codes are optimal in the sense they attain the maximal minimum b-distance and thus have the best possible capability against errors in b-symbol read channels. We show that there exists a linear MDS b-symbol code once one finds a suitable matrix. And then we introduce a method using projective geometry, which allows us to construct linear MDS b-symbol codes with a large range of lengths. As a result, we construct the following families of linear MDS b-symbol codes over finite fields.

(1) There exists an MDS $(n, 7)_q$ 3-symbol code for q being a prime power and $7 \leq n \leq q^3 + q^2 + q + 1$ (see Theorem 3.8).
(2) There exists an MDS $(n, 9)_q$ 4-symbol code for $q \geq 3$ being a prime power and $9 \leq n \leq q^4 + q^3 + q^2 + q + 1$ (see Theorem 3.10).
(3) There exists an MDS $(n, 2b+1)_q$ b-symbol code for q being a prime power, $q \geq b \geq 5$ and $2b + 1 \leq n \leq q^b - bq^{b-1} + \frac{b^2+3b}{2}$ (see Theorem 3.11).
(4) There exists an MDS $(n, 2b)_q$ b-symbol code with $n \geq 2b$ for $q \geq b - 1$ being a prime power, $b \geq 3$ or $q = 2, b = 4$ (see Theorem 3.13).
(5) There exists an MDS $(n, 10)_q$ 5-symbol code for $q \geq 3$ being a prime power and $n \geq 10$ (see Theorem 3.14).
(6) There exists an MDS $(\frac{q^{b+1}-1}{q-1}, 2b + 1)_q$ b-symbol code for q being prime power and any $b \geq 4$ (see Theorem 4.2).

We also propose the following two conjectures in Section 5.