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Varieties of Boolean inverse semigroups

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ABSTRACT

In an earlier work, the author observed that Boolean inverse semigroups, with semigroup homomorphisms preserving finite orthogonal joins, form a congruence-permutable variety of algebras, called *biases*. We give a full description of varieties of biases in terms of varieties of groups:

(1) Every free bias is residually finite. In particular, the word problem for free biases is decidable.
(2) Every proper variety of biases contains a largest finite symmetric inverse semigroup, and it is generated by its members that are monoids of generalized rook matrices over groups with zero.
(3) There is an order-preserving, one-to-one correspondence between proper varieties of biases and certain finite sequences of varieties of groups, descending in a strong sense defined in terms of wreath products by finite symmetric groups.

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Fully group-matricial
Radical
Congruence
Residually finite

1. Introduction

1.1. Motivation

Boolean inverse semigroups are an abstraction of semigroups of partial transformations which are closed under finite disjoint unions, originally studied by Tarski's school (cf. [26]). These objects have been for the last decade an active topic of research, see for example [13,15–18]. By definition, an inverse semigroup S with zero is Boolean if its semilattice of idempotents is (generalized) Boolean and S has finite orthogonal joins. (We refer to Section 2 for precise definitions.)

Unlike classes of structures such as groups, inverse semigroups, modules, rings, Lie algebras, the class of Boolean inverse semigroups is not defined as a variety of algebras in the sense of universal algebra: while the multiplication and the inversion of an inverse semigroup are *full* operations, orthogonal join is only a *partial* operation. The author introduced in [29, § 3.2] two full operations $\oslash$ and $\triangledown$ (cf. (2.1) and (2.2) for precise definitions), defined on every Boolean inverse semigroup in terms of multiplication, inversion, and the partial operation of orthogonal join, such that the semigroup homomorphisms preserving $\oslash$ and $\triangledown$ are exactly the *additive semigroup homomorphisms*; by definition, a semigroup homomorphism is additive if it preserves all finite orthogonal joins. Moreover, Boolean inverse semigroups can be characterized *via* a finite system of identities in the similarity type $(0, {}^{-1}, \cdot, \oslash, \triangledown)$. The models of those identities are called *biases*. The category of biases with bias homomorphisms is isomorphic to the category of Boolean inverse semigroups with additive semigroup homomorphisms. The author also proved in [29, Theorem 3.4.11] that the variety of all biases is *congruence-permutable*, which makes Boolean inverse semigroups much closer, in spirit, to groups and rings than to semigroups.

At the 2016 Workshop on New Directions in Inverse Semigroups in Ottawa, and then at the subsequent International Conference on Semigroups and Automata in Lisbon, the following programme was suggested by Mark Lawson:

Study varieties of Boolean inverse semigroups.

(By definition, a *variety* — or *equational class* — is the class of all structures, of a given similarity type, that satisfy a given set of identities.)

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