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Ding-Iohara algebras and quantum vertex algebras

Haisheng Li[a,b1], Shaobin Tan[b2] and Qing Wang[b3]
[a]Department of Mathematical Sciences
Rutgers University, Camden, NJ 08102, USA
[b]School of Mathematical Sciences, Xiamen University, Xiamen 361005, China

Abstract

In this paper, we associate quantum vertex algebras to a certain family of associative algebras $\widetilde{\mathcal{A}}(g)$ which are essentially Ding-Iohara algebras. To do this, we introduce another closely related family of associative algebras $\mathcal{A}(h)$. The associated quantum vertex algebras are based on the vacuum modules for $\mathcal{A}(h)$, whereas ϕ-coordinated modules for these quantum vertex algebras are associated to $\widetilde{\mathcal{A}}(g)$-modules. Furthermore, we classify their irreducible ϕ-coordinated modules.

1 Introduction

This paper is from a vertex algebra point of view to study a family of associative algebras $\widetilde{\mathcal{A}}(g)$ with $g(z)$ a rational function such that $g(z)g(1/z) = 1$. By definition, $\widetilde{\mathcal{A}}(g)$ is the associative unital algebra over $\mathbb{C}$, generated by

$$E_n,\ F_n,\ \Psi_n\ \ (n \in \mathbb{Z}),$$

subject to relations

$$\begin{aligned}
&E(z)E(w) = \iota_{w,z}(g(w/z))E(w)E(z), \quad F(z)F(w) = \iota_{w,z}(g(z/w))F(w)F(z),\\
&\Psi(z)E(w) = \iota_{w,z}(g(w/z))E(w)\Psi(z), \quad \Psi(z)F(w) = \iota_{w,z}(g(z/w))F(w)\Psi(z),\\
&[E(z), F(w)] = \delta\left(\frac{w}{z}\right)\Psi(w), \quad [\Psi(z), \Psi(w)] = 0,
\end{aligned}$$

where

$$E(z) = \sum_{n\in\mathbb{Z}} E_n z^{-n}, \quad F(z) = \sum_{n\in\mathbb{Z}} F_n z^{-n}, \quad \Psi(z) = \sum_{n\in\mathbb{Z}} \Psi_n z^{-n}.$$

Note that in the special case with

$$g(z) = \prod_{j=1,2,3} \frac{1 - q_j^{-1} z}{1 - q_j z},$$

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