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Number of common roots and resultant of two tropical univariate polynomials

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Abstract

It is well known that for two univariate polynomials over complex number field the number of their common roots is equal to the order of their resultant. In this paper, we show that this fundamental relationship still holds for the tropical polynomials under suitable adaptation of the notion of order, if the roots are simple and non-zero.

Keywords. tropical semifield; tropical resultant; common roots

MSC. 15A15, 15A80, 12K10.

1 Introduction

The resultant plays a crucial role in algebra and algebraic geometry [27, 23, 17, 5, 1, 9]. Let m, n be fixed. Let

$$\mathbf{A} = \mathbf{a}_0\mathbf{x}^m + \mathbf{a}_1\mathbf{x}^{m-1} + \cdots + \mathbf{a}_m \in \mathbb{C}[\mathbf{a}, \mathbf{x}]$$
$$\mathbf{B} = \mathbf{b}_0\mathbf{x}^n + \mathbf{b}_1\mathbf{x}^{n-1} + \cdots + \mathbf{b}_n \in \mathbb{C}[\mathbf{b}, \mathbf{x}]$$

Then the *resultant* $R \in \mathbb{C}[\mathbf{a}, \mathbf{b}]$ is defined as the smallest monic polynomial (w.r.t. a given order) such that, for every $a \in \mathbb{C}^{m+1}$ and $b \in \mathbb{C}^{n+1}$, if the two polynomials $\mathbf{A}(a, \mathbf{x}), \mathbf{B}(b, \mathbf{x}) \in \mathbb{C}[\mathbf{x}]$ have a common complex root then $R(a, b) = 0$.[1] We recall the following two well known fundamental properties of resultants. Let $a \in \mathbb{C}^{m+1}$ and $b \in \mathbb{C}^{n+1}$ such that $a_0, b_0 \neq 0$. Then we have

P1. The point (a, b) is a root of R if and only if the polynomials $\mathbf{A}(a, \mathbf{x})$ and $\mathbf{B}(b, \mathbf{x})$ have a common complex root. (Of course, the 'if' part is immediate from the definition and thus the interesting part is the 'only if').

P2. The order of the point (a, b) at R is equal to the number of common complex roots of the polynomials $\mathbf{A}(a, \mathbf{x})$ and $\mathbf{B}(b, \mathbf{x})$. (See the appendix)

[1] It is well-known that the resultant R can be defined in various other ways: for instance, in terms of Sylvester matrix, Bezout matrix, Barnett matrix, Hankel matrix (see [7] for a nice summary). Those definitions are more useful for computational purposes. However they are also more complicated when deducing theoretical or structural properties. Since the main interest of this paper is not computational but structural, we chose the more structural definition.