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Double extensions of Lie superalgebras in characteristic 2 with nondegenerate invariant supersymmetric bilinear form

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A B S T R A C T

A Lie (super)algebra with a non-degenerate invariant symmetric bilinear form will be called a NIS-Lie (super)algebra. The double extension of a NIS-Lie (super)algebra is the result of simultaneously adding to it a central element and an outer derivation so that the larger algebra has also a NIS. Affine loop algebras, Lie (super)algebras with symmetrizable Cartan matrix over any field, Manin triples, symplectic reflection (super)algebras are among the Lie (super)algebras suitable to be doubly extended.

We consider double extensions of Lie superalgebras in characteristic 2, and concentrate on peculiarities of these notions related with the possibility for the bilinear form, the center, and the derivation to be odd. Two Lie superalgebras we discovered by this method are indigenous to the characteristic 2.

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1. Introduction

1.1. NIS-algebras

Hereafter a Lie algebra with a **Non-degenerate Invariant Symmetric** bilinear form B will be called a *NIS-Lie algebra*.[2] If the Lie algebra is simple and finite-dimensional over $\mathbb{C}$, then such a form B is induced by the trace in any irreducible module of dimension > 1 and is a multiple of the Killing form induced by the trace in the adjoint representation.

The Killing form, however, becomes degenerate if the simple Lie algebra is defined over a field of characteristic $p > 0$, see [33], or on simple Lie superalgebras, see [14]. Some simple Lie algebras over the ground field of characteristic $p > 0$ (and some simple Lie superalgebras over any field) have no non-degenerate invariant bilinear form induced by the (super)-trace in any irreducible representation.

1.2. Double extensions of NIS-algebras

From ancient time people computed nontrivial central extensions and outer derivations of Lie (super)algebras separately, see a recent review [9]. There were known, however, important examples when both an outer derivation and a nontrivial central extension of a given Lie (super)algebra are present simultaneously; interestingly, this happens often in presence of a non-degenerate invariant (super)symmetric bilinear form.

Medina and Revoy in [30] introduced the notion of *double extensions* for NIS-Lie algebras in characteristic 0, and showed that any such algebra $\mathfrak{g}$ can be described inductively in terms of another NIS-Lie algebra $\mathfrak{a}$ of dimension $\dim(\mathfrak{g}) - 2$, provided the center of $\mathfrak{g}$ is not trivial (see also [20]). Solvable NIS-Lie algebras can be embraced by this method since the center is not trivial.

More interesting examples, however, are Manin triples, Manin–Olshansky triples, and affine Kac–Moody Lie algebras $\mathfrak{g}(A)$ with Cartan matrix A. Each of these examples is a double extension. For example, $\mathfrak{g}(A)$ is a double extension[3] of the loop algebra $\mathfrak{a} = \mathfrak{k} \otimes \mathbb{C}[t^{-1}, t]$ that has no Cartan matrix. The basis of $\mathfrak{a}$ lacks 2 elements of $\mathfrak{g}(A)$: the central element x and the outer derivative $D = t\frac{d}{dt}$ of $\mathfrak{a}$, where $t = e^{i\varphi}$, and φ is the angle parameter on the circle.

[2] Initially, the term *quadratic* was used to single out these algebras, see [1], [2], [4], [3], [5]. The term had been already occupied: V. Drinfeld [18] introduced *quadratic algebras* — the ones with quadratic relations — in a paper published in 1986 (and never translated from Russian, as far as we know); Yu. Manin used Drinfeld's term in his book, [29]. To avoid confusion, we decided from now on to call the algebras we are studying after their main properties. Besides, the form whose properties are vital for us is **bilinear**, not quadratic.

[3] The central extension in this case is defined by a 2-cocycle not of the form $B_{\mathfrak{a}}(D(a), b)$ as described by Eq. (1).