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RIGID IDEALS BY DEFORMING QUADRATIC LETTERPLACE IDEALS

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Abstract. We compute the deformation space of quadratic letterplace ideals $L(2, P)$ of finite posets P when its Hasse diagram is a rooted tree. These deformations are unobstructed. The deformed family has a polynomial ring as the base ring. The ideal $J(2, P)$ defining the full family of deformations is a rigid ideal and we compute it explicitly. In simple example cases $J(2, P)$ is the ideal of maximal minors of a generic matrix, the Pfaffians of a skew-symmetric matrix, and a ladder determinantal ideal.

1. Introduction

Monomial ideal theory has much developed into a branch of its own. But before that one studied polynomial ideals in general. Monomial ideals came about since they are specializations, typically initial ideals, of such ideals. One should then ask: Can monomial ideal theory give something back? Can one *start* with monomial ideals and derive interesting classes of polynomial ideals in general? Yes one can, and here we do this for a reasonably large class of monomial ideals. We get a full understanding of the polynomial ideals which specialize to the monomial ideals we start out from.

The ideals we work with. More precisely we consider quadratically generated letterplace ideals $L(2, P)$ associated to a finite poset P. These are precisely the edge ideals of Cohen-Macaulay bipartite graphs. Its generators are the monomials $x_{1,p}x_{2,q}$ where $p \leq q$ in the poset P. The fact that edge ideals of Cohen-Macaulay bipartite graphs have this form, is an astonishing discovery of J. Herzog and T. Hibi [14]. This class of ideals was generalized in [10] and further studied and generalized in [11] were they were called letterplace ideals, see Section 2.

Results. When the Hasse diagram of P has the form of a rooted tree we get a complete algebraic understanding of all ideals which are deformations of the quadratic letterplace ideals $L(2, P)$. This is all the more unusual and surprising for the following reason: Monomial ideals are degenerations of polynomial ideals. Thus whenever a monomial ideal is on a Hilbert scheme, it tends to be a singular point on the Hilbert scheme. Its infinitesimal deformations are then obstructed and it is a very hard and messy task to compute the space of all deformations.

However for the letterplace ideals $L(2, P)$ we consider, it turns out that every nice thing one could wish for, actually happens:

- The ideals $L(2, P)$ are *unobstructed*, i.e. every infinitesimal deformation lifts. In particular whenever this ideal is on a Hilbert scheme, it is a smooth point.
- The full deformation space which a priori is defined only over a complete local ring, actually lifts to a deformation over a *polynomial ring*. This follows from our

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