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Simple subquotients of big parabolically induced representations of p-adic groups

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ABSTRACT

This note is motivated by the problem of "uniqueness of supercuspidal support" in the modular representation theory of p-adic groups. We show that any counterexample to the same property for a finite reductive group lifts to a counterexample for the corresponding unramified p-adic group. To this end, we need to prove the following natural property: any simple subquotient of a parabolically induced representation is isomorphic to a subquotient of the parabolic induction of some simple subquotient of the original representation. The point is that we put no finiteness assumption on the original representation.

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1. Main results

Let $\mathbb{G}$ be a connected reductive group over a p-adic field F and put $G = \mathbb{G}(F)$. Let R be a Noetherian ring where p is invertible. We denote by $\mathrm{Rep}_R(G)$ the category of all smooth RG-modules. If $P = \mathbb{P}(F)$ is a parabolic subgroup of G with Levi quotient $M = \mathbb{M}(F)$, we denote by $i_P : \mathrm{Rep}_R(M) \longrightarrow \mathrm{Rep}_R(G)$ the parabolic induction functor and by $r_P : \mathrm{Rep}_R(G) \longrightarrow \mathrm{Rep}_R(M)$ its left adjoint, the Jacquet functor. Recall that a smooth RG-module is called *cuspidal* if it is killed by all proper Jacquet functors. We will prove the following result.

Theorem 1.1. *Let V be any smooth RM-module and let π be a* ***simple*** *and* ***admissible*** *RG-subquotient of $i_P(V)$. Assume that* ***one*** *of the following holds.*

(1) *V has level* 0*,* ***or***
(2) *Bernstein's second adjunction holds for* (G, R).

Then there is a simple smooth RM-module σ such that π is isomorphic to a subquotient of $i_P(\sigma)$. Moreover, if V is cuspidal, σ can be chosen to be a subquotient of V.

Of course the result is well-known if V has finite length, and the point is that we don't require any finiteness property on V here. We haven't been able to find a general argument that would work for any functor between abelian categories with suitable assumptions (such as commuting with all limits and colimits). Instead, our argument turns out to be quite intricate, so that we thought it might be interesting to advertise the result and its proof.

Let us explain the hypothesis.

(1) "*V has level* 0" means that $V = \sum_x V^{G_x^+}$ where x runs over vertices of the semisimple Bruhat–Tits building of G and G_x^+ is the pro-radical of the associated parahoric group. The objects of level 0 form a direct summand subcategory $\mathrm{Rep}_R^0(G)$ of $\mathrm{Rep}_R(G)$, and we have $i_P(\mathrm{Rep}_R^0(M)) \subset \mathrm{Rep}_R^0(G)$ and $r_P(\mathrm{Rep}_R^0(G)) \subset \mathrm{Rep}_R^0(M)$, see [2, Prop. 6.3 ii)].
(2) By the sentence "*Bernstein's second adjunction holds for* (G, R)", we mean that for any pair $(Q, \overline{Q})$ of opposed parabolic subgroups of any Levi subgroup M of G, the functor $\delta_Q.r_{\overline{Q}}$ is right adjoint to the functor i_Q. Here δ_Q is the modulus character of Q. By [2, Thm. 1.5] this property holds at least when G is a classical or linear group. Moreover, by [2, Prop. 6.3], this property also holds if we restrict these functors to the level 0 categories, without any hypothesis on G.

Let us now explain the typical application of this result that we have in mind. Suppose further that $\mathbb{G}$ extends to a connected reductive group over the integers $\mathcal{O}_F$ and that $\mathbb{P}$ extends to a parabolic subgroup over $\mathcal{O}_F$ (in other words, fix an hyperspecial point