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# Minimal generation of transitive permutation groups

Gareth M. Tracey*

*Mathematics Institute, University of Warwick,*
*Coventry CV4 7AL, United Kingdom*

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**Abstract**

This paper discusses upper bounds on the minimal number of elements $d(G)$ required to generate a transitive permutation group $G$, in terms of its degree $n$, and its order $|G|$. In particular, we reduce a conjecture of L. Pyber on the number of subgroups of the symmetric group $\mathrm{Sym}(n)$. We also prove that our bounds are best possible.

## 1 Introduction

A well-developed branch of finite group theory studies properties of certain classes of permutation groups as functions of their degree. The purpose of this paper is to study the minimal generation of transitive permutation groups.

For a group $G$, let $d(G)$ denote the minimal number of elements required to generate $G$. In [21], [7], [26] and [28], it is shown that $d(G) = O(n/\sqrt{\log n})$ whenever $G$ is a transitive permutation group of degree $n \geq 2$ (here, and throughout this paper, "log" means log to the base 2). A beautifully constructed family of examples due to L. Kovács and M. Newman shows that this bound is 'asymptotically best possible' (see Example 6.10), thereby ending the hope that a bound of $d(G) = O(\log n)$ could be proved.

The constants involved in these theorems, however, were never estimated. We prove:

**Theorem 1.1.** *Let $G$ be a transitive permutation group of degree $n \geq 2$. Then*

*(1)* $d(G) \leq \left\lfloor \frac{cn}{\sqrt{\log n}} \right\rfloor$*,where* $c := 1512660\sqrt{\log(2^{19}15)}/(2^{19}15) = 0.920581\ldots$*, and;*

*(2)* $d(G) \leq \left\lfloor \frac{c_1 n}{\sqrt{\log n}} \right\rfloor$*, where* $c_1 := \sqrt{3}/2 = 0.866025\ldots$*, unless each of the following conditions hold:*

- *(i)* $n = 2^k v$*, where* $v = 5$ *and* $17 \leq k \leq 26$*, or* $v = 15$ *and* $15 \leq k \leq 35$*, and;*
- *(ii) $G$ contains no soluble transitive subgroups.*

In fact, we prove a slightly stronger version of Theorem 1.1, which is given as Theorem 5.3. The following corollary is immediate.

*Electronic address: G.M.Tracey@warwick.ac.uk