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THE GALOIS CORRESPONDENCE THEOREM FOR GROUPOID ACTIONS

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Abstract. A theory about groupoids will be developed, emphasizing the notion of normal subgroupoid and quotient groupoid. It will be also constructed a version of the Galois correspondence theorem for groupoids acting on commutative rings.

1. Introduction

Groupoids were originally introduced by H. Brandt in [3]. Briefly, we say that a groupoid G is a small category whose morphisms are invertible. This categorical version can be found in [4], where are illustrated several examples and applications of groupoids.

The notion of partial (hence, also global) groupoid action was introduced by D. Bagio and A. Paques in [2], where it is interpreted as a generalization of partial group action (see [7] and [8]) as well as a generalization of partial ordered groupoid action (see [1]).

In Galois theory, many authors have left their contributions. S. U. Chase, D. K. Harrison and A. Rosenberg developed in [6] a Galois theory for commutative extensions $S \supset R$. Among the main results of that paper, Theorem 1.3 gives several equivalent conditions for the definition of a Galois extension and Theorem 2.3 states a one-to-one correspondence between the subgroups of the group G and the R-subalgebras which are separable and G-strong, where G is a finite group of ring automorphisms of S. For the particular case in which the ring S (and then R) has no idempotents but 0 and 1 the theorem gives a one-to-one correspondence between the subgroups of G and all the separable subalgebras of S. Shortly after, in [12], O. E. Villamayor and D. Zelinsky developed a Galois theory for rings with finitely many idempotents, requiring hypotheses on R and S and not on a prescribed group of automorphisms, using groupoids as an intermediary relation to get a bijection between all the separable subalgebras and some subgroups of the automorphisms group (the so called "fat"). That relation between subalgebras and groupoids anticipates the relations presented in this paper, although the authors did not formalize the definition of groupoid action.

The goal of this paper is to develop a version of the Galois correspondence theorem for the case of groupoids acting on commutative rings. The paper starts by introducing some preliminaries results about groupoids, groupoid actions, Galois extensions and separability in the section 2. In section 3 it will be presented the definition of normal subgroupoid and

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