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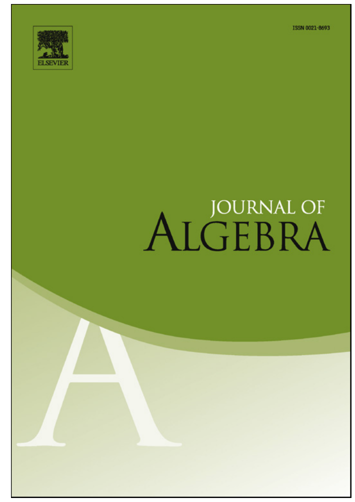

Symmetric cohomology of groups

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SYMMETRIC COHOMOLOGY OF GROUPS

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Abstract. We investigate the relationship between the symmetric, exterior and classical cohomologies of groups. The first two theories were introduced respectively by Staic and Zarelua. We show in particular, that there is a map from exterior cohomology to symmetric cohomology which is a split monomorphism in general and an isomorphism in many cases, but not always. We introduce two spectral sequences which help to explain the relationship between these cohomology groups. As a sample application we obtain that symmetric and classical cohomologies are isomorphic for torsion free groups.

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1. Introduction

Let G be a group and M be a G-module. In order to better understand 3-algebras arising in lattice field theory [3], Staic defined a variant of group cohomology, which he denoted by $HS^*(G,M)$ and called *symmetric cohomology of groups* [6]. Some aspects of this theory were later extended by Singh [5] and Todea [9]. There is an obvious natural transformation from the symmetric cohomology to the classical Eilenberg-MacLane cohomology

$$\alpha^n : HS^n(G,M) \to H^n(G,M), \quad n \geq 0.$$

According to [6],[7], α^n is an isomorphism if $n = 0, 1$ and is a monomorphism for $n = 2$. By Corollary 2.3 in [7] we know that α^2 is an isomorphism if G has no elements of order two.

Ten years prior to this, Zarelua had also defined a version of group cohomology, denoted by $H^*_\lambda(G,M)$ and called *exterior cohomology of groups* [10]. It also comes together with a natural transformation

$$\beta^n : H^n_\lambda(G,M) \to H^n(G,M),$$

with similar properties. The exterior cohomology has the following striking property: if G is a finite group of order d, then $H^i_\lambda(G,M) = 0$ for all $i \geq d$.

The aim of this work is to obtain more information about homomorphisms α^* and β^*. We construct a natural transformation $\gamma^n : H^n_\lambda(G,M) \to HS^n(G,M)$ such that the following diagram commutes:

$$\begin{tikzcd} H^n_\lambda(G,M) \arrow[rr, "\gamma^n"] \arrow[dr, "\beta^n"'] & & HS^n(G,M) \arrow[dl, "\alpha^n"] \\ & H^n(G,M). & \end{tikzcd}$$

Our results in Section 3 show that the homomorphism $\gamma^n : H^n_\lambda(G,M) \to HS^n(G,M)$ is a split monomorphism in general, and an isomorphism in certain cases, namely if $0 \leq n \leq 4$, or M has no elements of order two. In general, γ^5 is not an isomorphism.

Our next results are related to the homomorphism $\beta^n : H^n_\lambda(G,M) \to H^n(G,M)$. We construct a spectral sequence for which β^n are edge homomorphisms, $n \geq 0$. As any first quadrant spectral sequence, it gives a 5-term exact sequence (see for example [8, Exercise 5.1.3]) which has the following form:

$$0 \to H^2_\lambda(G,M) \xrightarrow{\beta^2} H^2(G,M) \to \prod_{C_2 \subset G} H^2(C_2,M) \to H^3_\lambda(G,M) \xrightarrow{\beta^3} H^3(G,M).$$

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