

Accepted Manuscript

Symmetric cohomology of groups

Mariam Pirashvili

PII: S0021-8693(18)30330-2

DOI: <https://doi.org/10.1016/j.jalgebra.2018.05.020>

Reference: YJABR 16719

To appear in: *Journal of Algebra*

Received date: 21 November 2017

Please cite this article in press as: M. Pirashvili, Symmetric cohomology of groups, *J. Algebra* (2018), <https://doi.org/10.1016/j.jalgebra.2018.05.020>

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.



SYMMETRIC COHOMOLOGY OF GROUPS

MARIAM PIRASHVILI

ABSTRACT. We investigate the relationship between the symmetric, exterior and classical cohomologies of groups. The first two theories were introduced respectively by Staic and Zarelua. We show in particular, that there is a map from exterior cohomology to symmetric cohomology which is a split monomorphism in general and an isomorphism in many cases, but not always. We introduce two spectral sequences which help to explain the relationship between these cohomology groups. As a sample application we obtain that symmetric and classical cohomologies are isomorphic for torsion free groups.

AMS classification: 20J06 18G40.

1. INTRODUCTION

Let G be a group and M be a G -module. In order to better understand 3-algebras arising in lattice field theory [3], Staic defined a variant of group cohomology, which he denoted by $HS^*(G, M)$ and called *symmetric cohomology of groups* [6]. Some aspects of this theory were later extended by Singh [5] and Todea [9]. There is an obvious natural transformation from the symmetric cohomology to the classical Eilenberg-MacLane cohomology

$$\alpha^n : HS^n(G, M) \rightarrow H^n(G, M), \quad n \geq 0.$$

According to [6],[7], α^n is an isomorphism if $n = 0, 1$ and is a monomorphism for $n = 2$. By Corollary 2.3 in [7] we know that α^2 is an isomorphism if G has no elements of order two.

Ten years prior to this, Zarelua had also defined a version of group cohomology, denoted by $H_\lambda^n(G, M)$ and called *exterior cohomology of groups* [10]. It also comes together with a natural transformation

$$\beta^n : H_\lambda^n(G, M) \rightarrow H^n(G, M),$$

with similar properties. The exterior cohomology has the following striking property: if G is a finite group of order d , then $H_\lambda^i(G, M) = 0$ for all $i \geq d$.

The aim of this work is to obtain more information about homomorphisms α^* and β^* . We construct a natural transformation $\gamma^n : H_\lambda^n(G, M) \rightarrow HS^n(G, M)$ such that the following diagram commutes:

$$\begin{array}{ccc} H_\lambda^n(G, M) & \xrightarrow{\gamma^n} & HS^n(G, M) \\ & \searrow \beta^n & \swarrow \alpha^n \\ & H^n(G, M) & \end{array}$$

Our results in Section 3 show that the homomorphism $\gamma^n : H_\lambda^n(G, M) \rightarrow HS^n(G, M)$ is a split monomorphism in general, and an isomorphism in certain cases, namely if $0 \leq n \leq 4$, or M has no elements of order two. In general, γ^5 is not an isomorphism.

Our next results are related to the homomorphism $\beta^n : H_\lambda^n(G, M) \rightarrow H^n(G, M)$. We construct a spectral sequence for which β^n are edge homomorphisms, $n \geq 0$. As any first quadrant spectral sequence, it gives a 5-term exact sequence (see for example [8, Exercise 5.1.3]) which has the following form:

$$0 \rightarrow H_\lambda^2(G, M) \xrightarrow{\beta^2} H^2(G, M) \rightarrow \prod_{C_2 \subset G} H^2(C_2, M) \rightarrow H_\lambda^3(G, M) \xrightarrow{\beta^3} H^3(G, M).$$

The research was supported by the EPSRC grant EP/N014189/1 Joining the dots: from data to insight.

Download English Version:

<https://daneshyari.com/en/article/8895900>

Download Persian Version:

<https://daneshyari.com/article/8895900>

[Daneshyari.com](https://daneshyari.com)