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On congruence-semisimple semirings and the K_0-group characterization of ultramatricial algebras over semifields

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Abstract

In this paper, we provide a complete description of congruence-semisimple semirings and introduce the pre-ordered abelian Grothendieck groups $K_0(S)$ and $SK_0(S)$ of the isomorphism classes of the finitely generated projective and strongly projective S-semimodules, respectively, over an arbitrary semiring S. We prove that the SK_0-groups and K_0-groups are complete invariants of, *i.e.*, completely classify, ultramatricial algebras over a semifield F. Consequently, we show that the SK_0-groups completely characterize zerosumfree congruence-semisimple semirings.

Keywords: congruence-semisimple semiring, projective semimodule, strongly projective semimodule, K_0-group, ultramatricial algebra
2010 MSC: Primary 16Y60, 16E20, 18G05

1. Introduction

As is well-known (see, for example, [5]), projective modules play a fundamental role in developing algebraic K-theory which, in turn, has crucial outcomes in many areas of modern mathematics such as topology, geometry, number theory, functional analysis, etc. In short, algebraic K-theory is a study of groups of the isomorphism classes of algebraic objects, the first of which is $K_0(R)$, Grothendieck's group of the isomorphism classes of finitely generated projective R-modules, and that is used to create a sort of dimension for R-modules that

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