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Auslander–Reiten components of symmetric special biserial algebras

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ABSTRACT

We provide a combinatorial algorithm for constructing the stable Auslander–Reiten component containing a given indecomposable module of a symmetric special biserial algebra using only information from its underlying Brauer graph. We also show that the structure of the Auslander–Reiten quiver is closely related to the distinct Green walks of the Brauer graph and detail the relationship between the precise shape of the stable Auslander–Reiten components for domestic Brauer graph algebras and their underlying graph. Furthermore, we show that the specific component containing a given simple or indecomposable projective module for any Brauer graph algebra is determined by the edge in the Brauer graph associated to the module.

1. Introduction

Within the study of the representation theory of finite dimensional algebras, one of the primary aims is to understand the indecomposable modules of the algebra along with the morphisms between them. The Auslander–Reiten quiver of an algebra is a means of presenting this information.

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The topic of this paper is on the Auslander–Reiten quiver of a class of algebras known as Brauer graph algebras, which coincides with the class of symmetric special biserial algebras ([13], [14]). They are finite dimensional algebras constructed from a decorated ribbon graph. The study of these algebras originates from the work of Richard Brauer on the modular representation theory of finite groups. Brauer graph algebras have since been studied extensively by various authors (see for example in [9], [11], [12], [13]).

It is known that Brauer graph algebras are of tame representation type ([1], [2], [16]) and those of finite representation type are precisely the Brauer tree algebras. The underlying Brauer graph of a domestic symmetric special biserial algebra has been described in [4]. It is also known that the indecomposable non-projective modules of a Brauer graph algebra are given by either string modules or band modules ([16]). The irreducible morphisms between indecomposable modules are then given by adding or deleting hooks and cohooks to strings ([5], [15]).

One of the interesting properties of Brauer graph algebras is that it is possible to read off some of the representation theory of the algebra from it's underlying Brauer graph. For example, a useful tool in representation theory is the projective resolution of a module. However, projective resolutions in algebras are difficult to calculate in general. For Brauer graph algebras, one can avoid such calculations and easily read off the projective resolutions of certain modules from the Brauer graph. These are given by the Green walks of the Brauer graph, which were first described in detail in [10] for Brauer trees and are shown to hold more generally in [13].

The aim of this paper is to study what information about the Auslander–Reiten quiver of a Brauer graph algebra we can read off from its underlying Brauer graph. There has already been extensive work on the Auslander–Reiten quiver of Brauer tree algebras. For example, a complete description of the Auslander–Reiten quiver of Brauer tree algebras, has been given in [8]. In [3], the location of the modules in the stable Auslander–Reiten quiver of a Brauer tree algebra has been described in terms of walks in the Brauer tree. However, the descriptions in both [3] and [8] do not address the case where the algebra is of infinite representation type, and thus, is associated to a Brauer graph that is not a Brauer tree.

In Section 3, we provide an algorithm for constructing the stable Auslander–Reiten component of a given string module of a Brauer graph algebra using only information from its underlying Brauer graph. This algorithm is of particular importance because it allows us to describe the string combinatorics of the algebra in terms of the Brauer graph. This algorithm thus allows us to prove several results later in the paper, which relate the number and shape of the components of the Auslander–Reiten quiver of the algebra to its underling Brauer graph.

In [7], the Auslander–Reiten components of self-injective special biserial algebras have been described. In particular, any Brauer graph algebra has a finite number of exceptional tubes in the stable Auslander–Reiten quiver. In Section 4, we show that the rank and the total number of these tubes is closely related to the distinct Green walks of the Brauer graph. Specifically, we prove the following.