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On a conjecture about Morita algebras

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ABSTRACT

We give an example of a Morita algebra A with a tilting module T such that the algebra $End_A(T)$ has dominant dimension at least two but is not a Morita algebra. This provides a counterexample to a conjecture by Chen and Xi from [5].

Introduction

In this article we assume that all rings are finite dimensional algebras over a field K and all modules are finitely generated right modules unless stated otherwise. Recall that the dominant dimension $domdim(M)$ of a module M with minimal injective coresolution (I^i) is defined as zero in case I^0 is not projective and $domdim(M) := \sup\{n \geq 0 | I^i$ is projective for $i = 0, 1, ..., n\} + 1$ otherwise. The dominant dimension of an algebra is defined as the dominant dimension of the regular module. It is well known that an algebra has dominant dimension at least one if and only if there is a minimal faith-

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ful projective–injective right module eA for some idempotent e of A. All Nakayama algebras have dominant dimension at least one and therefore have a minimal faithful projective–injective module given by the direct sum of all indecomposable projective–injective modules, see for example chapter 32 of [1]. For further information on the dominant dimension we refer to [10]. In [6] the authors introduced Morita algebras as algebras A that are algebras with dominant dimension at least two and a minimal faithful projective–injective module eA such that eAe is selfinjective. Morita algebras contain several important classes of algebras such as Schur algebras $S(n,r)$ for $n \geq r$ or blocks of category $\mathcal{O}$ and provide a useful generalisation of selfinjective algebras. At the end of the article [5] the authors provided three conjectures related to the dominant dimension of algebras. Their third conjecture states the following:

Conjecture. *Suppose two algebras A and B are derived equivalent. If A is a Morita algebra and the dominant dimension of B is at least two then also B is a Morita algebra.*

In [5] several special cases of this conjecture were proven. In this article we give a counterexample to this conjecture.

Theorem. *Let A be the Nakayama algebra with Kupisch series [4,5,4,5] with vertices numbered from 0 to 3. Let M be the module $e_0A \oplus e_1A \oplus e_3A \oplus e_1A/e_1J^4$. Then A is a Morita algebra and M is a tilting module of projective dimension two such that the algebra $B := End_A(M)$ is an algebra of dominant dimension equal to 4 that is not a Morita algebra.*

Note that B is derived equivalent to A, since endomorphism algebras of tilting modules are derived equivalent to the original algebra. Therefore, our theorem gives a counterexample to the conjecture. We found the counterexample to the conjecture while experimenting with the GAP-package QPA, see [8]. We thank Hongxing Chen and Changchang Xi for useful discussions in Stuttgart and Changchang Xi for proofreading and useful suggestions. We are thankful to the anonymous referee for many useful comments.

1. Proof of the theorem

In this section we give a proof of the theorem that we group into several smaller lemmas. We assume that the reader is familiar with the basics of the representation theory of finite dimensional algebras as explained for example in [3] or [2]. We use the conventions of [2]. Thus we use right modules and write arrows in quiver algebras from left to right. For background on Nakayama algebras and how to calculate projective or injective resolutions for modules in such algebras we refer to [7]. All algebras will be given by quiver and relations and are connected. Recall that the Kupisch series of a Nakayama algebra is just the sequence $[a_0, a_1, ..., a_r]$ when a_i denotes the dimension of