Accepted Manuscript

Separating Invariants of Finite Groups

Fabian Reimers

PII: S0021-8693(18)30214-X
DOI: https://doi.org/10.1016/j.jalgebra.2018.03.022
Reference: YJABR 16642

To appear in: *Journal of Algebra*

Received date: 17 March 2017

Please cite this article in press as: F. Reimers, Separating Invariants of Finite Groups, *J. Algebra* (2018), https://doi.org/10.1016/j.jalgebra.2018.03.022

SEPARATING INVARIANTS OF FINITE GROUPS

FABIAN REIMERS

Abstract. This paper studies separating invariants of finite groups acting on affine varieties through automorphisms. Several results, proved by Serre, Dufresne, Kac-Watanabe and Gordeev, and Jeffries and Dufresne exist that relate properties of the invariant ring or a separating subalgebra to properties of the group action. All these results are limited to the case of linear actions on vector spaces. The goal of this paper is to lift this restriction by extending these results to the case of (possibly) non-linear actions on affine varieties.

Under mild assumptions on the variety and the group action, we prove that polynomial separating algebras can exist only for reflection groups. The benefit of this gain in generality is demonstrated by an application to the semigroup problem in multiplicative invariant theory.

Then we show that separating algebras which are complete intersections in a certain codimension can exist only for 2-reflection groups. Finally we prove that a separating set of size $n + k - 1$ (where n is the dimension of X) can exist only for k-reflection groups.

Several examples show that most of the assumptions on the group action and the variety that we make cannot be dropped.

Introduction

Invariant theory studies the ring of those regular functions on an affine variety X that are constant on the orbits of an action of a linear algebraic group G on X, where the action is given by a morphism $G \times X \to X$. This paper considers the case where G is a finite group. We will always assume that the ground field K, over which G and X are defined, is algebraically closed. The affine variety X need not be irreducible. We refer to this setting by calling X a G-variety.

The group action on X induces an action on the coordinate ring $K[X]$ via $\sigma \cdot f := f \circ \sigma^{-1}$ for $\sigma \in G$ and $f \in K[X]$. The elements fixed by this action are called invariants and they form a subalgebra of $K[X]$:

$$K[X]^G := \{f \in K[X] \mid \sigma \cdot f = f \text{ for all } \sigma \in G\},$$

which is called the invariant ring. Although $K[X]^G$ is finitely generated as a K-algebra by a classical theorem of Noether [30], its minimal number of generators can be very large even for small groups and low-dimensional representations $X = V$ of G (see e.g. [19, Table in Section 5]). Derksen and Kemper [7] introduced the notion of separating invariants as a weaker concept than generating invariants. A subset $S \subseteq K[X]^G$ of the invariant ring is called *separating* if for all $x, y \in X$ the following holds: If there exists an invariant $f \in K[X]^G$ with $f(x) \neq f(y)$, then there exists an element $g \in S$ with $g(x) \neq g(y)$. Let γ_{sep} denote the smallest integer

Date: March 14, 2017.

2000 *Mathematics Subject Classification.* 13A50.

Key words and phrases. Invariant theory, separating invariants, reflection groups, multiplicative invariant theory.