Accepted Manuscript

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M. De Falco, F. de Giovanni, L.A. Kurdachenko, C. Musella

PII: S0021-8693(18)30212-6
DOI: https://doi.org/10.1016/j.jalgebra.2018.03.020
Reference: YJABR 16640

To appear in: *Journal of Algebra*

Received date: 16 November 2017

Please cite this article in press as: M. De Falco et al., The Metanorm and its Influence on the Group Structure, *J. Algebra* (2018), https://doi.org/10.1016/j.jalgebra.2018.03.020

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The Metanorm and its Influence on the Group Structure[1]

M. De Falco – F. de Giovanni – L.A. Kurdachenko – C. Musella

Abstract

The *norm* of a group was introduced by R. Baer as the intersection of all normalizers of subgroups, and it was later proved that the norm is always contained in the second term of the upper central series of the group. The aim of this paper is to study the influence on a group G of the behaviour of its *metanorm* $M(G)$, defined as the intersection of all normalizers of non-abelian subgroups of G. The metanorm is related to the so-called *metahamiltonian groups*, i.e. groups in which all non-abelian subgroups are normal, introduced by Romalis and Sesekin fifty years ago. Among other results, it is proved here that if G is any locally finite group whose metanorm is metabelian but not nilpotent, then G is metahamiltonian (or equivalently $M(G) = G$), unless the order of the commutator subgroup of $M(G)$ is the square of a prime number.

Mathematics Subject Classification (2010): 20F24, 20F16, 20F50
Keywords: norm; metanorm; metahamiltonian group

1 Introduction

In 1934, Reinhold Baer [1] defined the *norm* $N(G)$ of a group G to be the intersection of normalizers of all subgroups of G. Of course, the norm of any group G contains the centre $Z(G)$, and a group coincides with its norm if and only if all its subgroups are normal. It was proved by Baer that $N(G)$ is abelian if G is not periodic and that $N(G) = Z(G)$ whenever $N(G)$ contains elements of infinite order. Later, Schenkman [23] showed that $N(G)$ is always contained in the second term $\zeta_2(G)$ of the upper central series of G. This latter result is also a direct consequence of Cooper's theorem on centrality of power automorphisms (see [3]), since an element belongs to the norm of a group if and only if it acts by conjugation as a power automorphism.

The concept of norm was recently generalized by restricting the attention just to normalizers of certain subgroups (see [2]). In fact, if $\mathfrak{X}$ is any group class, the $\mathfrak{X}$-*norm* of a group G is the intersection of all normalizers of subgroups of G which are not in $\mathfrak{X}$. In particular, if $\mathfrak{I}$ is the class consisting only

[1]The first, second and fourth authors are members of GNSAGA (INdAM), and work within the ADV-AGTA project

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