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# Quantizations of multiplicative hypertoric varieties at a root of unity



Iordan Ganev

*Institute of Science and Technology Austria, Am Campus 1, 3400 Klosterneuburg, Austria*

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## ABSTRACT

We construct quantizations of multiplicative hypertoric varieties using an algebra of  $q$ -difference operators on affine space, where  $q$  is a root of unity in  $\mathbb{C}$ . The quantization defines a matrix bundle (i.e. Azumaya algebra) over the multiplicative hypertoric variety and admits an explicit finite étale splitting. The global sections of this Azumaya algebra is a hypertoric quantum group, and we prove a localization theorem. We introduce a general framework of Frobenius quantum moment maps and their Hamiltonian reductions; our results shed light on an instance of this framework.

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*E-mail address:* [iordan.ganev@ist.ac.at](mailto:iordan.ganev@ist.ac.at).

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## 1. Introduction

In this paper, we use  $q$ -difference operators to construct a class of Azumaya algebras on multiplicative hypertoric varieties, for  $q$  a root of unity in  $\mathbb{C}$ . We identify their algebras of global sections with central reductions of hypertoric quantum groups. This construction leads to a description of representations of the hypertoric quantum group at a root of unity as coherent sheaves on the multiplicative hypertoric variety, via an analogue of the Beilinson–Bernstein localization theorem.

Our results can be understood as an instance of the paradigm that Azumaya algebras descend to Azumaya algebras under Hamiltonian reduction. This paradigm is formalized by the notion of Frobenius quantum moment maps and their Hamiltonian reductions, resulting in a framework that captures the appropriate level of generality to construct and study Azumaya algebras over a swath of classical moduli spaces constructed by group-valued Hamiltonian reduction. This framework encompasses: (1) the present setting of multiplicative hypertoric varieties, (2) Hilbert schemes and double affine Hecke algebras [28], and (3) quantum  $D$ -modules on flag varieties [2] – each of which lead to Azumaya algebras over their classical degenerations – and also (4) multiplicative quiver varieties [18], where such results are expected. Frobenius quantum Hamiltonian reduction is also a natural quantum/multiplicative analogue of the framework of [7] in positive characteristic, deepening the rich parallels between the representation theory of quantum groups at roots of unity and universal enveloping algebras in positive characteristic.

We briefly describe the notion of a Frobenius quantum moment map. Let  $G$  be a reductive algebraic group over  $\mathbb{C}$  with Lie algebra  $\mathfrak{g}$  and let  $q$  be a primitive  $\ell$ -th root of unity. Let  $U_q\mathfrak{g}$  denote the (unrestricted) quantum group and let  $\mathcal{O}_q(G) \subset U_q\mathfrak{g}$  denote the left coideal subalgebra of ad-locally finite elements.<sup>1</sup> The algebra  $\mathcal{O}_q(G)$  contains a central subalgebra  $\mathcal{O}_q^{(\ell)}(G)$  isomorphic to the algebra  $\mathcal{O}(G)$  of functions on  $G$ . Let

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<sup>1</sup> Hence,  $\mathcal{O}_q(G)$  denotes the ad-equivariant quantized coordinate algebra, as opposed to the bi-equivariant ‘FRT’ algebra.

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