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Triangulated equivalence between a homotopy category and a triangulated quotient category

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Abstract: Given two complete hereditary cotorsion pairs $(\mathcal{Q}, \mathcal{R})$ and $(\mathcal{Q}', \mathcal{R}')$ in a bicomplete abelian category \mathcal{G} such that $\mathcal{Q}' \subseteq \mathcal{Q}$ and $\mathcal{Q} \cap \mathcal{R} = \mathcal{Q}' \cap \mathcal{R}'$, Becker showed that there exists a hereditary abelian model structure $\mathcal{M} = (\mathcal{Q}, \mathcal{W}, \mathcal{R}')$ on \mathcal{G} , where \mathcal{W} is a thick subcategory of \mathcal{G} . We prove that the homotopy category $\text{Ho}(\mathcal{M})$ of \mathcal{M} is triangulated equivalent to the triangulated quotient category $D^b(\mathcal{G})_{[\widehat{\mathcal{Q}, \mathcal{R}'}]} / K^b(\mathcal{Q}' \cap \mathcal{R}')$, where $D^b(\mathcal{G})_{[\widehat{\mathcal{Q}, \mathcal{R}'}]}$ is the subcategory of $D^b(\mathcal{G})$ consisting of all homology bounded complexes with both finite \mathcal{Q} dimension and \mathcal{R}' dimension and $K^b(\mathcal{Q}' \cap \mathcal{R}')$ is the bounded homotopy category of $\mathcal{Q}' \cap \mathcal{R}'$ (core) objects. Applications are given in the category of modules. It is shown that the homotopy category of the Gorenstein flat (resp., Ding projective and Gorenstein AC-projective) model structure on the category of modules established by Gillespie and his coauthors can be realized as a certain triangulated quotient category.

Keywords: hereditary abelian model structure; homotopy category; triangulated equivalence; singularity category.

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1. Introduction

In 2002, Hovey [32] introduced the notion of abelian model categories as bicomplete abelian categories possessing a compatible model structure. The most wonderful result in [32], which is now known as *Hovey's correspondence*, says that there is a one-to-one correspondence between abelian model structures and complete cotorsion pairs. More specifically, an abelian model structure on a bicomplete abelian category \mathcal{G} is equivalent to a triple $(\mathcal{Q}, \mathcal{W}, \mathcal{R}')$ of subcategories in \mathcal{G} such that \mathcal{W} is thick and $(\mathcal{Q}, \mathcal{W} \cap \mathcal{R}')$ and $(\mathcal{Q} \cap \mathcal{W}, \mathcal{R}')$ form two complete cotorsion pairs, where \mathcal{W} (resp., \mathcal{Q} and \mathcal{R}') is the subcategory of trivial (resp., cofibrant and fibrant) objects associated to the corresponding abelian model structure. We refer the reader to [32, Theorem 2.2] for more details.

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