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Triangulated equivalence between a homotopy category and a triangulated quotient category

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Abstract: Given two complete hereditary cotorsion pairs $(\mathcal{Q},\mathcal{R})$ and $(\mathcal{Q}',\mathcal{R}')$ in a bicomplete abelian category $\mathcal{G}$ such that $\mathcal{Q}'\subseteq\mathcal{Q}$ and $\mathcal{Q}\cap\mathcal{R}=\mathcal{Q}'\cap\mathcal{R}'$, Becker showed that there exists a hereditary abelian model structure $\mathcal{M}=(\mathcal{Q},\mathcal{W},\mathcal{R}')$ on $\mathcal{G}$, where $\mathcal{W}$ is a thick subcategory of $\mathcal{G}$. We prove that the homotopy category $\mathrm{Ho}(\mathcal{M})$ of $\mathcal{M}$ is triangulated equivalent to the triangulated quotient category $D^{\mathrm{b}}(\mathcal{G})_{\widehat{[\mathcal{Q},\mathcal{R}']}}/K^{\mathrm{b}}(\mathcal{Q}'\cap\mathcal{R}')$, where $D^{\mathrm{b}}(\mathcal{G})_{\widehat{[\mathcal{Q},\mathcal{R}']}}$ is the subcategory of $D^{\mathrm{b}}(\mathcal{G})$ consisting of all homology bounded complexes with both finite $\mathcal{Q}$ dimension and $\mathcal{R}'$ dimension and $K^{\mathrm{b}}(\mathcal{Q}'\cap\mathcal{R}')$ is the bounded homotopy category of $\mathcal{Q}'\cap\mathcal{R}'$ (core) objects. Applications are given in the category of modules. It is shown that the homotopy category of the Gorenstein flat (resp., Ding projective and Gorenstein AC-projective) model structure on the category of modules established by Gillespie and his coauthors can be realized as a certain triangulated quotient category.

Keywords: hereditary abelian model structure; homotopy category; triangulated equivalence; singularity category.

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1. Introduction

In 2002, Hovey [32] introduced the notion of abelian model categories as bicomplete abelian categories possessing a compatible model structure. The most wonderful result in [32], which is now known as *Hovey's correspondence*, says that there is a one-to-one correspondence between abelian model structures and complete cotorsion pairs. More specifically, an abelian model structure on a bicomplete abelian category $\mathcal{G}$ is equivalent to a triple $(\mathcal{Q},\mathcal{W},\mathcal{R}')$ of subcategories in $\mathcal{G}$ such that $\mathcal{W}$ is thick and $(\mathcal{Q},\mathcal{W}\cap\mathcal{R}')$ and $(\mathcal{Q}\cap\mathcal{W},\mathcal{R}')$ form two complete cotorsion pairs, where $\mathcal{W}$ (resp., $\mathcal{Q}$ and $\mathcal{R}'$) is the subcategory of trivial (resp., cofibrant and fibrant) objects associated to the corresponding abelian model structure. We refer the reader to [32, Theorem 2.2] for more details.

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