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Twisted modules and co-invariants for commutative vertex algebras of jet schemes

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TWISTED MODULES AND CO-INVARIANTS FOR COMMUTATIVE VERTEX ALGEBRAS OF JET SCHEMES

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ABSTRACT. Let $Z \subset \mathbb{A}^k$ be an affine scheme over $\mathbb{C}$ and $\mathcal{J}Z$ its jet scheme. It is well-known that $\mathbb{C}[\mathcal{J}Z]$, the coordinate ring of $\mathcal{J}Z$, has the structure of a commutative vertex algebra. This paper develops the orbifold theory for $\mathbb{C}[\mathcal{J}Z]$. A finite-order linear automorphism g of Z acts by vertex algebra automorphisms on $\mathbb{C}[\mathcal{J}Z]$. We show that $\mathbb{C}[\mathcal{J}^g Z]$, where $\mathcal{J}^g Z$ is the scheme of g–twisted jets has the structure of a g-twisted $\mathbb{C}[\mathcal{J}Z]$ module. We consider spaces of orbifold coinvariants valued in the modules $\mathbb{C}[\mathcal{J}^g Z]$ on orbicurves $[Y/G]$, with Y a smooth projective curve and G a finite group, and show that these are isomorphic to $\mathbb{C}[Z^G]$.

1. INTRODUCTION

Let $Z \subset \mathbb{A}^k$ be an affine scheme over $\mathbb{C}$, and

$$\mathcal{J}Z := \mathrm{Hom}_{Sch}(\mathrm{Spec}\, \mathbb{C}[[t]], Z)$$

its jet scheme. It is well-known [4, 3] that the coordinate ring $\mathbb{C}[\mathcal{J}Z]$ has the structure of a commutative vertex algebra. Such vertex algebras often arise as quasiclassical limits of noncommutative vertex algebras, and have found a number of applications, such as in the study of chiral differential operators and the invariant theory of vertex algebras [1, 2, 8]. This paper is devoted to the orbifold theory of the commutative vertex algebra $\mathbb{C}[\mathcal{J}Z]$, or more specifically, to the construction of twisted modules for $\mathbb{C}[\mathcal{J}Z]$ and coinvariants valued in such.

Given a linear automorphism $g : Z \to Z$ of finite order m, we obtain an induced action on $\mathcal{J}Z$ and hence on $\mathbb{C}[\mathcal{J}Z]$ by vertex algebra automorphisms. We may also associate to this data the *g–twisted jet scheme*

$$\mathcal{J}^g Z := \{x(t^{1/m}) \in \mathrm{Hom}(\mathrm{Spec}\, \mathbb{C}[[t^{1/m}]], Z) | x(e^{2\pi i/m} t^{1/m}) = g(x(t^{1/m}))\}$$

of g-equivariant jets. An abbreviated version of our result is the following :