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Effective Subgroup Separability of Finitely Generated Nilpotent Groups

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Abstract

This paper studies effective separability for subgroups of finitely generated nilpotent groups and more broadly effective subgroup separability of finitely generated nilpotent groups. We provide upper and lower bounds that are polynomial with respect to the logarithm of the word length for infinite index subgroups of nilpotent groups. In the case of normal subgroups, we provide an exact computation generalizing work of the second author. We introduce a function that quantifies subgroup separability, and we provide polynomial upper and lower bounds. We finish by demonstrating that our results extend to virtually nilpotent groups and stating some open questions.

1 Introduction

Let G be a finitely generated group with a subgroup H. We say that H is a *separable subgroup* if for each $g \in G \setminus H$ there exists a group morphism to a finite group $\pi : G \to Q$ such that $\pi(g) \notin \pi(H)$. If the trivial subgroup is separable, we say G is *residually finite*. The group G is called *subgroup separable*, also known in the literature as locally extended residually finite (LERF), if every finitely generated subgroup of G is separable. Subgroup separability is thus a natural generalization of residual finiteness.

The study of subgroup separability in the literature has been to understand which groups satisfy these properties. For instance, closed surface groups, free groups, fundamental groups of geometric 3-manifolds, finitely generated nilpotent groups, and polycyclic groups have all been shown to be subgroup separable and subsequently, residually finite in [1, 7, 12, 15, 16, 18]. Recently, there is a lot of interesting in making effective various separability properties such as residual finiteness and subgroup separability. For a finitely generated group G with a finite generating subset S and a finitely generated subgroup $H \leq G$, we introduce a function $\mathrm{F}_{G,H,S}(n)$ on the natural numbers that quantifies the separability of H in G. In particular, the value $\mathrm{F}_{G,H,S}(n)$ on a natural number n is such that every element in the complement of H of word length at most n can be distinguished in

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