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Bases for local Weyl modules for the hyper and truncated current $\mathfrak{sl}_2$-algebras

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A B S T R A C T

We use the theory of Gröbner–Shirshov bases for ideals to construct linear bases for graded local Weyl modules for the (hyper) current and the truncated current algebras associated to the finite-dimensional complex simple Lie algebra $\mathfrak{sl}_2$. The main result is a characteristic-free construction of bases for this important family of modules for the hyper current $\mathfrak{sl}_2$-algebra. In the positive characteristic setting this work represents the first construction in the literature. In the characteristic zero setting, the method provides a different construction of the Chari–Pressley (also Kus–Littelmann) bases and the Chari–Venkatesh bases for local Weyl modules for the current $\mathfrak{sl}_2$-algebra. Our construction allows us to obtain new bases for the local Weyl modules for truncated current $\mathfrak{sl}_2$-algebras with very particular properties.

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0. Introduction

The category of level zero representations of affine (and quantum affine) algebras and its full subcategory of finite-dimensional representations have been extensively studied in the last three decades. The representation theory of these algebras gives a significant contribution to identifying interesting families of finite-dimensional representations of *loop* and *current algebras* (cf. [9]), such as the universal finite-dimensional highest weight modules called *local Weyl modules*, which became objects of independent and deep interest in the last ten years (cf. [5–8,12,13,16,24,25]). Among the generalizations of current algebras (as in [6]) special attention is given to the *truncated current algebras* (cf. [11,20,30]), which are finite-dimensional quotients of the current algebra.

During the last decade, the study of the positive characteristic analogues of these local Weyl modules for current and loop algebras was also developed in [2,3,15], where we can see that the characteristic zero and positive characteristic cases share several properties. To differentiate the positive characteristic case for the current algebra, we will refer to it as *hyper current algebra*.

Even with a large number of papers dedicated to the study of structure, character, decomposition, tensor product, fusion product, and reducibility of Weyl modules, only few involve the construction of bases for these modules (cf. [8–10,20]). Additionally, some recent papers (cf. [26–28]) focused in studying properties of the bases constructed in [8,9].

A basis for local Weyl modules for the current $\mathfrak{sl}_2$-algebra was first constructed by Chari–Pressley [9] and it was used by Chari–Loktev [8] in the construction of a basis for local Weyl modules for current algebras associated to $\mathfrak{sl}_{n+1}$, $n > 1$. Recently, two constructions came up: Kus–Littelmann [20] constructed a basis for truncated local Weyl modules for the current $\mathfrak{sl}_2$-algebra whose construction contemplates the Chari–Pressley basis for graded local Weyl modules by using a very different approach; and Chari–Venkatesh [10] provided the construction of a different basis for graded local Weyl modules for the current $\mathfrak{sl}_2$-algebra. Unfortunately, we still do not see how to adapt any of those constructions to the positive characteristic setting. This fact already appeared with many other results first proved in characteristic zero and then generalized to positive characteristic setting by using very different tools as in the pairs of papers [9,15] and [2,7].

The present paper was originally intended to present a different approach to obtain some new bases for graded local Weyl modules for the current $\mathfrak{sl}_2$-algebra. While this work was developed, we noticed that we could provide a characteristic-free construction of a basis for graded local Weyl modules for the current and hyper current $\mathfrak{sl}_2$-algebra. This work provides the first explicit construction of bases for hyper current algebras (see Corollary 5.3) and it is also an alternative construction to get the Chari–Pressley (cf. [9]) and Kus–Littelmann (cf. [20]) bases in the characteristic zero setting. Further, our method is also an alternative construction (see Theorem 6.1) to get Chari–Venkatesh basis (cf. [10]) with some advantages due its interpretation coming from monomial orders.