Journal of Algebra 505 (2018) 150–193

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Degenerate cyclotomic Hecke algebras and higher level Heisenberg categorification

Marco Mackaay [a,*], Alistair Savage [b]

[a] *Departamento de Matemática, Universidade do Algarve and CAMGSD, Instituto Superior Técnico, Portugal*
[b] *Department of Mathematics and Statistics, University of Ottawa, Canada*

ARTICLE INFO

Article history:
Received 5 February 2018
Available online 28 March 2018
Communicated by Volodymyr Mazorchuk

MSC:
primary 20C08
secondary 18D10, 19A22

Keywords:
Categorification
Heisenberg algebra
Hecke algebras
Cyclotomic quotients
Diagrammatic calculus

ABSTRACT

We associate a monoidal category $\mathcal{H}^\lambda$ to each dominant integral weight λ of $\widehat{\mathfrak{sl}}_p$ or $\mathfrak{sl}_\infty$. These categories, defined in terms of planar diagrams, act naturally on categories of modules for the degenerate cyclotomic Hecke algebras associated to λ. We show that, in the $\mathfrak{sl}_\infty$ case, the level d Heisenberg algebra embeds into the Grothendieck ring of $\mathcal{H}^\lambda$, where d is the level of λ. The categories $\mathcal{H}^\lambda$ can be viewed as a graphical calculus describing induction and restriction functors between categories of modules for degenerate cyclotomic Hecke algebras, together with their natural transformations. As an application of this tool, we prove a new result concerning centralizers for degenerate cyclotomic Hecke algebras.

Contents

1. Introduction 151
2. The diagrammatic category 153
3. A filtration and the associated graded category 163
4. The Grothendieck ring 166

* Corresponding author.
E-mail addresses: mmackaay@ualg.pt (M. Mackaay), alistair.savage@uottawa.ca (A. Savage).
URL: http://alistairsavage.ca (A. Savage).

https://doi.org/10.1016/j.jalgebra.2018.03.004
0021-8693/

5. Degenerate cyclotomic Hecke algebras 170
6. Action on categories of modules for degenerate cyclotomic Hecke algebras 178
7. Properties of the action functors 186
8. Further directions 190
References 192

1. Introduction

The Heisenberg algebra plays a fundamental role in many areas of mathematics and physics. The universal enveloping algebra of the infinite-dimensional Heisenberg Lie algebra is the associative algebra with generators $p_n^{\pm}$, $n \in \mathbb{N}_+$, and c, and relations

$$p_n^+ p_m^+ = p_m^+ p_n^+, \quad p_n^- p_m^- = p_m^- p_n^-, \quad p_n^+ p_m^- = p_m^- p_n^+ + \delta_{n,m} c, \quad p_n^{\pm} c = c p_n^{\pm}, \quad n, m \in \mathbb{N}_+.$$

(See Section 4.1 for a more detailed treatment.) On any irreducible representation, the central element c acts by a constant. For a positive integer d, the (associative) Heisenberg algebra $\mathfrak{h}_d$ of level d is the quotient of this algebra by the ideal generated by $c - d$.

In [16], Khovanov introduced a diagrammatic monoidal category $\mathcal{H}$ that acts naturally on categories of modules for symmetric groups. He proved that the Grothendieck ring of $\mathcal{H}$ contains the level one Heisenberg algebra $\mathfrak{h}_1$ and conjectured that the two are actually equal. This work has inspired an active area of research into Heisenberg categorification. Replacing group algebras of symmetric groups by Hecke algebras of type A led to the q-deformed categorification of [21], while replacing them by wreath product algebras led to categorifications of (quantum) lattice Heisenberg algebras in [10,25].

One remarkable feature of Khovanov's category is that degenerate affine Hecke algebras H_n appear naturally in the endomorphism spaces of certain objects. The group algebras of symmetric groups are level one cyclotomic quotients of these degenerate affine Hecke algebras, wherein the polynomial generators of H_n are mapped to the Jucys–Murphy elements. It is thus natural to conjecture that suitably modified versions of Khovanov's category should act on categories of modules for more general degenerate cyclotomic Hecke algebras. These modified categories should categorify higher level Heisenberg algebras and should encode much of the representation theory of degenerate affine Hecke algebras and their cyclotomic quotients.

On the other hand, cyclotomic Hecke algebras and their degenerate versions have appeared in the context of categorification before. In particular, Ariki's categorification theorem ([1,5]) relates the representation theory of these algebras to highest weight irreducible representations of affine Lie algebras of type A. In addition, Brundan–Kleshchev ([3,4]) and Rouquier ([24]) have related cyclotomic Hecke algebras to the quiver Hecke algebras appearing in the Khovanov–Lauda–Rouquier categorification of quantum $\mathfrak{sl}_n$ ([17,24]).

In the current paper we define a family of diagrammatic $\Bbbk$-linear monoidal categories $\mathcal{H}^{\lambda}$ depending on a dominant integral weight λ of $\mathfrak{sl}_\infty$ (when $\Bbbk$ is of characteristic zero)