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Axiomatic theory of Burnside rings. (I) ☆

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we study abstract Burnside rings of essentially finite categories. Under unique epi-mono factorization property and the existence of coequalizers for some kind, we prove the existence of a fundamental exact sequence for ABR. Furthermore, an ABR can be embedded into a direct product of rational character rings.

0. Introduction

0.1. Witt construction

In order to categorically formulate the theory of Burnside rings of finite groups, we start with Witt ring construction. If a set Ω is equipped with a map $\varphi : \Omega \longrightarrow \widetilde{\Omega}$ into an algebraic system $\widetilde{\Omega}$ satisfying the following conditions:

(a) φ is injective.
(b) the image of φ is a subalgebra of $\widetilde{\Omega}$,

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then the set Ω becomes an algebra system by which φ is an algebra homomorphism. Constructing an algebra Ω by this way is called **Witt construction**. The homomorphism φ is called a **ghost map**. The algebra $\widetilde{\Omega}$ is called a **ghost algebra**.

Example 0.1 *(Witt ring).* The most famous example of a Witt construction is the Witt ring. Let $W(\mathbb{Z})$ be the set of sequences $x = (x_1, x_2, \cdots)$ of integers. For each $n \in \mathbb{Z}$, define a map

$$w_n : W(\mathbb{Z}) \longrightarrow \mathbb{Z}; x \mapsto \sum_{d|n} d x_d^{n/d},$$

and then we have a map into the product ring

$$w = (w_n) : W(\mathbb{Z}) \longrightarrow \mathbb{Z}^{\mathbb{N}}; x \mapsto (w_n(x)).$$

As is well known $W(\mathbb{Z})$ has a ring structure called the Witt ring with injective ring homomorphisms w. See [10] and [11].

Example 0.2 *(λ-ring).* The set $t\mathbb{Z}[[t]]$ of power series without constant term is a commutative ring with multiplication defined by Hadamard product

$$\left(\sum_{n\geq 1} a_n t^n\right) * \left(\sum_{n\geq 1} b_n t^n\right) = \left(\sum_{n\geq 1} a_n b_n t^n\right).$$

Let $\Lambda(\mathbb{Z}) := 1 + t\mathbb{Z}[[t]]$ be the set of unitary power series in $\mathbb{Z}$. Then the map

$$L : \Lambda(\mathbb{Z}) \longrightarrow t\mathbb{Z}[[t]]; a(t) \mapsto t\frac{d}{dt}\log a(t)$$

is an injective and its image is a subring of $t\mathbb{Z}[[t]]$. The resulting Witt construction gives a λ-ring.

This ring is isomorphic to the ring $W(\mathbb{Z})$ of universal Witt vectors and also to the complete Burnside ring $\widehat{\Omega}(\widehat{C})$ of the infinite cyclic group $\widehat{C}$. These rings are all defined by Witt construction: