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Dedicated to Idun Reiten on the occasion of her 75th birthday

Abstract

A finite-dimensional algebra A over an algebraically closed field K is called periodic if it is periodic under the action of the syzygy operator in the category of A-A-bimodules. The periodic algebras are self-injective and occurred naturally in the study of tame blocks of group algebras, actions of finite groups on spheres, hypersurface singularities of finite Cohen-Macaulay type, and Jacobian algebras of quivers with potentials. Recently, the tame periodic algebras of polynomial growth have been classified and it is natural to attempt to classify all tame periodic algebras. We introduce the weighted surface algebras of triangulated surfaces with arbitrarily oriented triangles and describe their basic properties. In particular, we prove that all these algebras, except the singular tetrahedral algebras, are symmetric tame periodic algebras of period 4. Moreover, we describe the socle deformations of the weighted surface algebras and prove that all these algebras are also symmetric tame periodic algebras of period 4. The main results of this paper form an important step towards a classification of all periodic symmetric tame algebras of non-polynomial growth, and lead to a complete description of all algebras of generalized quaternion type with 2-regular Gabriel quivers [36].

Keywords: Syzygy, Periodic algebra, Self-injective algebra, Surface algebra, Tame algebra
2010 MSC: 16D50, 16E30, 16G20, 16G60, 16G70

1. Introduction and the main results

Throughout this paper, K will denote a fixed algebraically closed field. By an algebra we mean an associative finite-dimensional K-algebra with an identity. For an algebra A, we denote by $\operatorname{mod} A$ the category of finite-dimensional right A-modules and by D the standard duality $\operatorname{Hom}_K(-, K)$ on $\operatorname{mod} A$. An algebra A is called *self-injective* if A_A is injective in $\operatorname{mod} A$, or equivalently, the projective modules in $\operatorname{mod} A$ are injective. A prominent class of self-injective algebras is formed by the *symmetric algebras* A for which there exists an associative, non-degenerate symmetric K-bilinear form $(-,-) : A \times A \to K$. Classical examples of symmetric algebras are provided by the blocks of group algebras of finite groups and the Hecke algebras of finite Coxeter groups. In fact, any algebra A is the quotient algebra of its trivial extension algebra $\mathrm{T}(A) = A \ltimes D(A)$, which is a symmetric algebra. Two self-injective algebras A and Λ are said to be *socle equivalent* if the quotient algebras $A/\operatorname{soc}(A)$ and $\Lambda/\operatorname{soc}(\Lambda)$ are isomorphic.

From the remarkable Tame and Wild Theorem of Drozd (see [17, 23]) the class of algebras over K may be divided into two disjoint classes. The first class consists of the *tame algebras* for which the indecomposable modules occur in each dimension d in a finite number of discrete and a finite number of one-parameter families. The second class is

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