# Idempotents of double Burnside algebras, $L$-enriched bisets, and decomposition of $p$-biset functors

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# Idempotents of double Burnside algebras, $L$-enriched bisets, and decomposition of $p$-biset functors

**Serge Bouc**

**Abstract:** Let $R$ be a (unital) commutative ring, and $G$ be a finite group with order invertible in $R$. We introduce new idempotents $\epsilon_{T,S}^G$ in the double Burnside algebra $RB(G,G)$ of $G$ over $R$, indexed by conjugacy classes of minimal sections $(T,S)$ of $G$ (i.e. sections such that $S \leq \Phi(T)$). These idempotents are orthogonal, and their sum is equal to the identity. It follows that for any biset functor $F$ over $R$, the evaluation $F(G)$ splits as a direct sum of specific $R$-modules indexed by minimal sections of $G$, up to conjugation.
The restriction of these constructions to the biset category of $p$-groups, where $p$ is a prime number invertible in $R$, leads to a decomposition of the category of $p$-biset functors over $R$ as a direct product of categories $\mathcal{F}_L$ indexed by *atoric* $p$-groups $L$ up to isomorphism.
We next introduce the notions of *$L$-enriched biset* and *$L$-enriched biset functor* for an arbitrary finite group $L$, and show that for an atoric $p$-group $L$, the category $\mathcal{F}_L$ is equivalent to the category of $L$-enriched biset functors defined over elementary abelian $p$-groups.
Finally, the notion of *vertex* of an indecomposable $p$-biset functor is introduced (when $p \in R^\times$), and when $R$ is a field of characteristic different from $p$, the objects of the category $\mathcal{F}_L$ are characterized in terms of vertices of their composition factors.

**AMS subject classification:** 18B99, 19A22, 20J15

**Keywords:** Minimal sections, idempotents, double Burnside algebra, enriched biset functor, atoric

## 1. Introduction

Let $R$ denote throughout a commutative ring (with identity element). For a finite group $G$, we consider the double Burnside algebra $RB(G,G)$ of $G$ over $R$. In the case where the order of $G$ is invertible in $R$, we introduce idempotents $\epsilon_{T,S}^G$ in $RB(G,G)$, indexed by the set $\mathcal{M}(G)$ of minimal sections of $G$, i.e. the set of pairs $(T,S)$ of subgroups of $G$ with $S \trianglelefteq T$ and $S \leq \Phi(T)$, where $\Phi(T)$ is the Frattini subgroup of $T$ (such sections have been considered in Section 5 of [9]). The idempotent $\epsilon_{T,S}^G$ only depends on the conjugacy class of $(T,S)$ in $G$. Moreover, the idempotents $\epsilon_{T,S}^G$, where $(T,S)$ runs through a set $[\mathcal{M}(G)]$ of representatives of orbits of $G$ acting on $\mathcal{M}(G)$ by conjugation, are orthogonal, and their sum is equal to the identity element of $RB(G,G)$.