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Idempotents of double Burnside algebras, L -enriched bisets, and decomposition of p -biset functors

Serge Bouc

Abstract: Let R be a (unital) commutative ring, and G be a finite group with order invertible in R . We introduce new idempotents $\epsilon_{T,S}^G$ in the double Burnside algebra $RB(G, G)$ of G over R , indexed by conjugacy classes of minimal sections (T, S) of G (i.e. sections such that $S \leq \Phi(T)$). These idempotents are orthogonal, and their sum is equal to the identity. It follows that for any biset functor F over R , the evaluation $F(G)$ splits as a direct sum of specific R -modules indexed by minimal sections of G , up to conjugation.

The restriction of these constructions to the biset category of p -groups, where p is a prime number invertible in R , leads to a decomposition of the category of p -biset functors over R as a direct product of categories $\mathcal{F}_L$ indexed by *atoric* p -groups L up to isomorphism.

We next introduce the notions of *L -enriched biset* and *L -enriched biset functor* for an arbitrary finite group L , and show that for an atoric p -group L , the category $\mathcal{F}_L$ is equivalent to the category of L -enriched biset functors defined over elementary abelian p -groups.

Finally, the notion of *vertex* of an indecomposable p -biset functor is introduced (when $p \in R^\times$), and when R is a field of characteristic different from p , the objects of the category $\mathcal{F}_L$ are characterized in terms of vertices of their composition factors.

AMS subject classification: 18B99, 19A22, 20J15

Keywords: Minimal sections, idempotents, double Burnside algebra, enriched biset functor, atoric

1. Introduction

Let R denote throughout a commutative ring (with identity element). For a finite group G , we consider the double Burnside algebra $RB(G, G)$ of G over R . In the case where the order of G is invertible in R , we introduce idempotents $\epsilon_{T,S}^G$ in $RB(G, G)$, indexed by the set $\mathcal{M}(G)$ of minimal sections of G , i.e. the set of pairs (T, S) of subgroups of G with $S \trianglelefteq T$ and $S \leq \Phi(T)$, where $\Phi(T)$ is the Frattini subgroup of T (such sections have been considered in Section 5 of [9]). The idempotent $\epsilon_{T,S}^G$ only depends on the conjugacy class of (T, S) in G . Moreover, the idempotents $\epsilon_{T,S}^G$, where (T, S) runs through a set $[\mathcal{M}(G)]$ of representatives of orbits of G acting on $\mathcal{M}(G)$ by conjugation, are orthogonal, and their sum is equal to the identity element of $RB(G, G)$.

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