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Maximal subsemigroups of finite transformation and diagram monoids

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ABSTRACT

We describe and count the maximal subsemigroups of many well-known transformation monoids, and diagram monoids, using a new unified framework that allows the treatment of several classes of monoids simultaneously. The problem of determining the maximal subsemigroups of a finite monoid of transformations has been extensively studied in the literature. To our knowledge, every existing result in the literature is a special case of the approach we present. In particular, our technique can be used to determine the maximal subsemigroups of the full spectrum of monoids of order- or orientation-preserving transformations and partial permutations considered by I. Dimitrova, V. H. Fernandes, and co-authors. We only present details for the transformation monoids whose maximal subsemigroups were not previously known; and for certain diagram monoids, such as the partition, Brauer, Jones, and Motzkin monoids.

The technique we present is based on a specialised version of an algorithm for determining the maximal subsemigroups of any finite semigroup, developed by the third and fourth authors, and available in the Semigroups package for GAP, an open source computer algebra system. This allows us to con-

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cisely present the descriptions of the maximal subsemigroups, and to clearly see their common features.

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1. Introduction, definitions, and summary of results

A proper subsemigroup of a semigroup S is *maximal* if it is contained in no other proper subsemigroup of S . Similarly, a proper subgroup of a group G is *maximal* if it is not contained in any other proper subgroup of G . If G is a finite group, then every non-empty subsemigroup of G is a subgroup, and so these notions are not really distinct in this case. The same is not true if G is an infinite group. For instance, the natural numbers form a subsemigroup, but not a subgroup, of the integers under addition.

Maximal subgroups of finite groups have been extensively studied, in part because of their relationship to primitive permutation representations, and, for example, the Frattini subgroup. The maximal subgroups of the finite symmetric groups are described, in some sense, by the O’Nan–Scott Theorem [38] and the Classification of Finite Simple Groups. Maximal subgroups of infinite groups have also been extensively investigated; see [3,4,7,8,32,37] and the references therein.

There are also many papers in the literature relating to maximal subsemigroups of semigroups that are not groups. We describe the finite case in more detail below; for the infinite case see [16] and the references therein. Maximal subgroups of infinite groups, and maximal subsemigroups of infinite semigroups, are very different from their finite counterparts. For example, there exist infinite groups with no maximal subgroups, infinite groups with as many maximal subgroups as subsets, and subgroups that are not contained in any maximal subgroup. Analogous statements hold for semigroups.

In [24], Graham, Graham, and Rhodes showed that every maximal subsemigroup of a finite semigroup has certain features, and that every maximal subsemigroup must be one of a small number of types. As is often the case for semigroups, this classification depends on the description of maximal subgroups of certain finite groups. In [13], Donovan, Mitchell, and Wilson describe an algorithm for calculating the maximal subsemigroups of an arbitrary finite semigroup, starting from the results in [24]. In the current paper, we use the framework provided by this algorithm to describe and count the maximal subsemigroups of several families of finite monoids of partial transformations and monoids of partitions. The maximal subsemigroups of many families of transformation monoids have already been described or counted, principally by I. Dimitrova, V. H. Fernandes, and co-authors; see [9,11,10,20,25] and the references therein. It is possible to recover the previously known results about transformation monoids using the approach we present, and, indeed, to illustrate the usefulness of our technique, we have included full details in a longer version of this paper; see [17].

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