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The center of the enveloping algebra of the p-Lie algebras $\mathfrak{sl}_n$, $\mathfrak{pgl}_n$, $\mathfrak{psl}_n$, when p divides n

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ABSTRACT

Let $\mathfrak{g} = \text{Lie}(G)$, be a reductive Lie algebra over an algebraically closed field F with char $F = p > 0$. Suppose G satisfies Jantzen's standard assumptions. Then the structure of Z, the center of the enveloping algebra $U(\mathfrak{g})$, is described by (the extended) Veldkamp's theorem. We examine here the deviation of Z from this theorem, in case $\mathfrak{g} = \mathfrak{sl}_n$, $\mathfrak{pgl}_n$ or $\mathfrak{psl}_n$ and $p|n$. It is shown that Veldkamp's description is valid for $\mathfrak{pgl}_n$. This implies that Friedlander–Parshall–Donkin decomposition theorem for $F[\mathfrak{g}]^{\mathfrak{g}}$ holds in case p is good for a semi-simple simply connected G (excluding, if $p = 2$, A_1-factors of G). In case $\mathfrak{g} = \mathfrak{sl}_n$ or $\mathfrak{g} = \mathfrak{psl}_n$ we prove a fiber product theorem for a polynomial extension of Z. However Veldkamp's description mostly fails for $\mathfrak{sl}_n$ and $\mathfrak{psl}_n$. In particular Z is not Cohen–Macaulay if $n > 4$, in both cases. Contrary to a result of Kac–Weisfeiler, we show for an odd prime p that $Z_p(U(\mathfrak{sl}_p))$ and $U(\mathfrak{sl}_p)^{SL_p}$ do not generate $Z(U(\mathfrak{sl}_p))$. We also show for $\mathfrak{sl}_n$ that the codimension of the non-Azumaya locus of Z is at least 2 (if $n \geq 3$), and exceeds 2 if $n > 4$. This refutes a conjecture of Brown–Goodearl. We then show that Z is factorial (excluding $\mathfrak{g} = \mathfrak{pgl}_2$), thus confirming a conjecture of Premet–Tange. We also verify Humphreys conjecture on the parametrization of blocks, in case p is good for G.

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1. Introduction

Let $\mathfrak{g} = \mathrm{Lie}(G)$ be the Lie algebra of a reductive connected algebraic group over an algebraically closed field F with char $F = p > 0$. We say that G satisfies the (Jantzen's) standard assumptions if:

(1) The derived group DG of G is simply connected,
(2) p is good for G,
(3) There exists a G-invariant non-degenerate bilinear form on $\mathfrak{g}$.

The structure of $Z(U(\mathfrak{g}))$ where $\mathfrak{g} = \mathrm{Lie}(G)$ and G is a reductive algebraic group satisfying the standard assumptions is known as "Veldkamp's theorem". It is a consequence of many contributions due to Veldkamp [45], Kac–Weisfeiler [31], DeConcini–Kac–Procesi [14], Brown–Gordon [10] and Mirkovic–Rumynin [36].

Let $Z_p := Z_p(U(\mathfrak{g}))$ denote the p-center of $U(\mathfrak{g})$. This is a polynomial ring. Let $U(\mathfrak{g})^G$ be the so called Harish–Chandra center, where G acts by the adjoint action on $U(\mathfrak{g})$ (extending the one on $\mathfrak{g}$). It is a consequence of Demazure theorem (and its extension due to Slodoway) that $U(\mathfrak{g})^G$ is a polynomial ring in rank($\mathfrak{g}$)-variables. The extended version of Veldkamp's theorem can be stated as follows:

Theorem A.

(1) *The fiber product theorem:*

$$Z_p \bigotimes_{Z_p^G} U(\mathfrak{g})^G \cong Z(U(\mathfrak{g})).$$

In particular $Z(U(\mathfrak{g}))$ is generated by the generators of the p-center and the Harish–Chandra center,

(2) *$Z(U(\mathfrak{g}))$ is a free Z_p-module of rank $p^{\mathrm{rank}(\mathfrak{g})}$. In particular $Z(U(\mathfrak{g}))$ is a complete intersection,*

(3) *similar statements hold for $S(\mathfrak{g})^{\mathfrak{g}}$ with relation to $S(\mathfrak{g})^G$ and $S_p(\mathfrak{g})$.*

Our goal here is to consider the remaining reductive cases when p is good for G. This amounts to G having direct summands of type A_{n-1} where $p|n$. We shall therefore concentrate on these semi-simple summands and on the related Lie algebras $\mathfrak{sl}_n$, $\mathfrak{pgl}_n$ and $\mathfrak{psl}_n$.

Our main results for $Z(U(\mathfrak{sl}_n))$ are as follows:

Theorem B. *Suppose $p|n$. Then there exists an order p winding automorphism ϕ of $U(\mathfrak{gl}_n)$ such that:*

$$Z_p(U(\mathfrak{gl}_n)) \bigotimes_{Z_p(U(\mathfrak{gl}_n))^{GL_n}} (U(\mathfrak{gl}_n)^{GL_n})^{\phi} \cong Z(U(\mathfrak{sl}_n))[e_{11}^p - e_{11}].$$