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A correction of the decomposability result in a paper by Meyer–Neutsch

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ABSTRACT

In their paper of 1993, Meyer and Neutsch established the existence of a 48-dimensional associative subalgebra in the Griess algebra $\mathfrak{G}$. By exhibiting an explicit counter example, the present paper shows a gap in the proof one of the key results in Meyer and Neutsch's paper, which states that an idempotent a in the Griess algebra is indecomposable if and only its Peirce 1-eigenspace (i.e. the 1-eigenspace of the linear transformation $L_a : x \mapsto ax$) is one-dimensional. The present paper fixes this gap, and shows a more general result: let V be a real commutative nonassociative algebra with an associative inner product, and let c be a nonzero idempotent of V such that its Peirce 1-eigenspace is a subalgebra; then, c is indecomposable if and only if its Peirce 1-eigenspace is one-dimensional. The proof of this result is based on a general variational argument for real commutative metrised algebras with inner product.

1. Introduction

In their 1993 paper [5], Meyer and Neutsch established the existence of a 48-dimensional associative subalgebra in the Griess algebra $\mathfrak{G}$ and conjectured that 48 is the

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largest possible dimension of an associative subalgebra of $\mathfrak{G}$. This conjecture has been proved by Miyamoto [6].

One of the key results claimed in [5], Theorem 11 asserts that an idempotent $c \in \mathfrak{G}$ is indecomposable if and only if the Peirce eigenspace $\mathfrak{G}_c(1)$ with the eigenvalue 1 is at most one-dimensional. This result was used by Miyamoto (see Theorem 6.8 in [6]) and in a later research on associative subalgebras of low-dimensional Majorana algebras, see for instance p. 576 in [2] and section 3.1 in [4].

Unfortunately, there is a gap in the argument in the proof of Theorem 11 in [5]. Recall the statement of the theorem.

Theorem 1.1 *(Theorem 11, in [5]). An idempotent $a \in \mathfrak{G}$ is indecomposable if and only if the Peirce eigenspace $\mathfrak{G}_a(1) = \{x \in \mathfrak{G} : ax = x\}$ is at most one-dimensional.*

Remark 1.2. Note also that $\mathfrak{G}_a(1)$ is a subalgebra in $\mathfrak{G}$, see [5].

More precisely, in their proof of the implication "$\dim \mathfrak{G}_c(1) \geq 2 \Rightarrow c$ is decomposable", Meyer and Neutsch use an erroneous argument that a (normalized) cubic form

$$\varphi(x) := \frac{\langle x,\, x^2\rangle}{\langle x,\, x\rangle^{\frac{3}{2}}} \tag{1}$$

provides at least two linearly independent idempotents. Let us explain what is a gap here. It is well known (and easy to see) that $x \neq 0$ is a stationary point of $\varphi(x)$ precisely when $x^2 = \lambda x$ for some $\lambda \in \mathbb{R}$. If $\lambda \neq 0$ then x gives rise to a (nonzero) idempotent by normalizing: indeed $c = x/\lambda$ implies $c^2 = c$. In order to get two distinct (linearly independent) idempotents, Meyer and Neutsch suggest that a minimum and a maximum point of ϕ would provide us with two linearly independent idempotents (see p. 15 in [5]). But, since ϕ is an *odd function*, this may give us two anti-collinear, and therefore *linearly dependent*, elements. In particular, it is clear from (1) that x is a local maximum of φ if and only if $-x$ is a local minimum of φ.

Remark 1.3. We will point out that the variational argument used by the authors in the proof of Lemma 2 and Theorem 8 in [5] causes no problems because one uses there an *even* function (based on a quartic form). This, together with nonzero lower estimate (76), allows one to guarantee at least two linear independent stationary points (corresponding to a local maximum and minimum).

The variational argument used in [5] would be easily fixed if one could prove that a general cubic form always has at least 2 linearly independent stationary points with