

Accepted Manuscript

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Ebrahim Zangoiezhadeh, Kazem Khashyarmanesh

PII: S0021-8693(18)30115-7

DOI: <https://doi.org/10.1016/j.jalgebra.2018.01.039>

Reference: YJABR 16575

To appear in: *Journal of Algebra*

Received date: 14 March 2016

Please cite this article in press as: E. Zangoiezhadeh, K. Khashyarmanesh, Some splitting theorems for extension and torsion functors of local cohomology modules, *J. Algebra* (2018), <https://doi.org/10.1016/j.jalgebra.2018.01.039>

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**SOME SPLITTING THEOREMS FOR EXTENSION AND
TORSION FUNCTORS OF LOCAL COHOMOLOGY MODULES**

EBRAHIM ZANGOIEZADEH¹ AND KAZEM KHASHYARMANESH^{2,*}

¹*Department of Mathematics, Payame Noor University,
P.O.Box 19395-3697, Tehran, Iran*

and

²*Department of Pure Mathematics, Ferdowsi University of Mashhad,
P.O.Box 1159-91775, Mashhad, Iran.*

ABSTRACT. Let R be a commutative ring with identity, \mathfrak{a} an ideal of R and M a finitely generated R -module. In this paper, for fixed integers t and j , we study the existence of the following isomorphisms of local cohomology modules:

- (i) $H_{\mathfrak{a}}^t(M/xM) \cong H_{\mathfrak{a}}^t(M) \oplus H_{\mathfrak{a}}^{t+1}(M)$;
 - (ii) $\text{Ext}_R^j(\frac{R}{\mathfrak{a}}, H_{\mathfrak{a}}^t(M/xM)) \cong \text{Ext}_R^j(\frac{R}{\mathfrak{a}}, H_{\mathfrak{a}}^t(M)) \oplus \text{Ext}_R^j(\frac{R}{\mathfrak{a}}, H_{\mathfrak{a}}^{t+1}(M))$; and,
 - (iii) $\text{Tor}_j^R(\frac{R}{\mathfrak{a}}, H_{\mathfrak{a}}^t(M/xM)) \cong \text{Tor}_j^R(\frac{R}{\mathfrak{a}}, H_{\mathfrak{a}}^t(M)) \oplus \text{Tor}_j^R(\frac{R}{\mathfrak{a}}, H_{\mathfrak{a}}^{t+1}(M))$,
- for some \mathfrak{a} -filter regular element x on M . Also, we provide some applications of the above isomorphisms.

1. INTRODUCTION

Let (R, \mathfrak{m}) be a commutative Noetherian local ring with non-zero identity and M be a finitely generated R -module. In the theory of generalized Cohen-Macaulay modules, for a parameter element x of M , there exists a sufficiently large n (dependent on the choice of the element x) such that

$$H_{\mathfrak{m}}^i(M/x^n M) \cong H_{\mathfrak{m}}^i(M) \oplus H_{\mathfrak{m}}^{i+1}(M)$$

for all $i < \dim M - 1$ (see [7]). For arbitrary ideal \mathfrak{a} of R and element $x \in \mathfrak{a}$, splitting of local cohomology module $H_{\mathfrak{a}}^i(M/xM)$ into local cohomology modules of M with support in \mathfrak{a} is a useful tool for studying properties of local cohomology modules (see [7, 1, 6, 4]). This leads to the following natural question (see also Question in [3]).

1.1. Let M be a finitely generated module over a Noetherian ring R and \mathfrak{a} an ideal of R . Let t be a non-negative integer. Does there exist an element $x \in \mathfrak{a}$ such that

$$H_{\mathfrak{a}}^t(M/xM) \cong H_{\mathfrak{a}}^t(M) \oplus H_{\mathfrak{a}}^{t+1}(M).$$

2000 *Mathematics Subject Classification.* 13D45, 13D07.

Key words and phrases. Local cohomology, Split exact sequence, Extension functor, Torsion functor, Filter regular element.

*Corresponding author.

E-mail addresses: e.zangoie@pnu.ac.ir and khashyar@ipm.ir.

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