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SOME SPLITTING THEOREMS FOR EXTENSION AND TORSION FUNCTORS OF LOCAL COHOMOLOGY MODULES

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Abstract. Let R be a commutative ring with identity, $\mathfrak{a}$ an ideal of R and M a finitely generated R-module. In this paper, for fixed integers t and j, we study the existence of the following isomorphisms of local cohomology modules:

(i) $H^t_{\mathfrak{a}}(M/xM) \cong H^t_{\mathfrak{a}}(M) \oplus H^{t+1}_{\mathfrak{a}}(M)$;
(ii) $\mathrm{Ext}^j_R(\frac{R}{\mathfrak{a}}, H^t_{\mathfrak{a}}(M/xM)) \cong \mathrm{Ext}^j_R(\frac{R}{\mathfrak{a}}, H^t_{\mathfrak{a}}(M)) \oplus \mathrm{Ext}^j_R(\frac{R}{\mathfrak{a}}, H^{t+1}_{\mathfrak{a}}(M))$; and,
(iii) $\mathrm{Tor}^R_j(\frac{R}{\mathfrak{a}}, H^t_{\mathfrak{a}}(M/xM)) \cong \mathrm{Tor}^R_j(\frac{R}{\mathfrak{a}}, H^t_{\mathfrak{a}}(M)) \oplus \mathrm{Tor}^R_j(\frac{R}{\mathfrak{a}}, H^{t+1}_{\mathfrak{a}}(M))$,

for some $\mathfrak{a}$-filter regular element x on M. Also, we provide some applications of the above isomorphisms.

1. Introduction

Let $(R, \mathfrak{m})$ be a commutative Noetherian local ring with non-zero identity and M be a finitely generated R-module. In the theory of generalized Cohen-Macaulay modules, for a parameter element x of M, there exists a sufficiently large n (dependent on the choice of the element x) such that

$$H^i_{\mathfrak{m}}(M/x^nM) \cong H^i_{\mathfrak{m}}(M) \oplus H^{i+1}_{\mathfrak{m}}(M)$$

for all $i < \dim M - 1$ (see [7]). For arbitrary ideal $\mathfrak{a}$ of R and element $x \in \mathfrak{a}$, splitting of local cohomology module $H^i_{\mathfrak{a}}(M/xM)$ into local cohomology modules of M with support in $\mathfrak{a}$ is a useful tool for studying properties of local cohomology modules (see [7, 1, 6, 4]). This leads to the following natural question (see also Question in [3]).

1.1. Let M be a finitely generated module over a Noetherian ring R and $\mathfrak{a}$ an ideal of R. Let t be a non-negative integer. Does there exist an element $x \in \mathfrak{a}$ such that

$$H^t_{\mathfrak{a}}(M/xM) \cong H^t_{\mathfrak{a}}(M) \oplus H^{t+1}_{\mathfrak{a}}(M).$$

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