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Artinian and noetherian partial skew groupoid rings

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Let $\alpha = \{\alpha_g : R_{g^{-1}} \to R_g\}_{g \in \mathrm{mor}(G)}$ be a partial action of a groupoid G on a (not necessarily associative) ring R and let $S = R \star_\alpha G$ be the associated partial skew groupoid ring. We show that if α is global and unital, then S is left (right) artinian if and only if R is left (right) artinian and $R_g = \{0\}$, for all but finitely many $g \in \mathrm{mor}(G)$. We use this result to prove that if α is unital and R is alternative, then S is left (right) artinian if and only if R is left (right) artinian and $R_g = \{0\}$, for all but finitely many $g \in \mathrm{mor}(G)$. This result applies to partial skew group rings, in particular. Both of the above results generalize a theorem by J. K. Park for classical skew group rings, i.e. the case when R is unital and associative, and G is a group which acts globally on R. We provide two additional applications of our main results. Firstly, we generalize I. G. Connell's classical result for group rings by giving a characterization of artinian (not necessarily associative) groupoid rings. This result is in turn applied to partial group algebras. Secondly, we give a characterization of artinian Leavitt path algebras. At the end of the article, we relate noetherian and artinian properties of partial skew groupoid rings to those of global skew groupoid rings, as well as establish two Maschke-type results, thereby generalizing

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results by M. Ferrero and J. Lazzarin for partial skew group rings to the case of partial skew groupoid rings.

1. Introduction

In 1963, I. G. Connell [8] showed that if R is an associative and unital ring, and G is a group, then the group ring $R[G]$ is left (right) artinian if and only if R is left (right) artinian and G is finite. Later on, D. S. Passman gave examples of artinian twisted group rings by infinite groups (see [37, Section 4]). Passman's examples show that Connell's result can not be generalized to twisted group rings or, more generally, crossed products.

Another type of crossed products, generalizing group rings, are the skew group rings. Recall that if $\alpha : G \ni g \mapsto \alpha_g \in \mathrm{Aut}(R)$ is a group homomorphism from G to $\mathrm{Aut}(R)$, the group of ring automorphisms of R, then the *skew group ring* $R *_\alpha G$ is the set of finite formal sums of the form $\sum_{g\in G} r_g g$ with addition defined componentwise and multiplication defined by the relations $(rg)(sh) = (r\alpha_g(s))gh$, for $r, s \in R$ and $g, h \in G$. In 1979, J. K. Park [36] generalized Connell's result to skew group rings by showing the following.

Theorem 1.1 *(Park [36]). If R is a unital and associative ring, and α is a group homomorphism from a group G to $\mathrm{Aut}(R)$, then the skew group ring $R *_\alpha G$ is left (right) artinian if and only if R is left (right) artinian and G is finite.*

In this article, we consider two generalizations (see Theorem 1.2 and Theorem 1.3) of Theorem 1.1 in the context of partial skew groupoid rings over non-associative rings, i.e. rings which are not necessarily associative. Previously, partial skew groupoid rings have been defined only over associative rings. However, since there are many interesting examples of non-associative rings with various types of actions, it is natural to seek such a theory in this more general sense. For instance, our Theorem 1.2 holds when R equals any of the algebras in the infinite chain of classical Cayley–Dickson doublings: the real numbers $\mathbb{R}$, the complex numbers $\mathbb{C}$, Hamilton's quaternions $\mathbb{H}$, Graves' octonions $\mathbb{O}$, the sedenions $\mathbb{S}$, the trigintaduonions $\mathbb{T}$ etc. Other important classes of examples to which our Theorem 1.2 can be applied comes from the cases when R is a Jordan algebra or a Baric algebra.

The notion of a partial action of a group on a C*-algebra was introduced by R. Exel [13], as an efficient tool to their study. Since then, the theory of (twisted) partial actions on C*-algebras has played a key role in the characterization of several classes of C*-algebras as crossed products by (twisted) partial actions, e.g. AF-algebras [15], Bunce–Deddens algebras [14], Cuntz–Krieger algebras [17] and Cuntz–Li algebras [7], (see also the survey [9]). In a purely algebraic context, partial skew group rings were