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On subsequences of quiddity cycles and Nichols algebras

M. Cuntz

Institut für Algebra, Zahlentheorie und Diskrete Mathematik, Fakultät für Mathematik und Physik, Leibniz Universität Hannover, Welfengarten 1, D-30167 Hannover, Germany

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ABSTRACT

We provide a tool to obtain local descriptions of quiddity cycles. As an application, we classify rank two affine Nichols algebras of diagonal type.

1. Introduction

Consider any structure counted by Catalan numbers, for example triangulations of a convex polygon by non-intersecting diagonals. Given such a triangulation (see for example Fig. 1), the sequence of numbers of triangles at each vertex has been called its *quiddity cycle* by Conway and Coxeter [2] because this sequence is the first row of any *frieze pattern* and thus contains the "quiddity" of the frieze. The main result of this paper is:

E-mail address: cuntz@math.uni-hannover.de.

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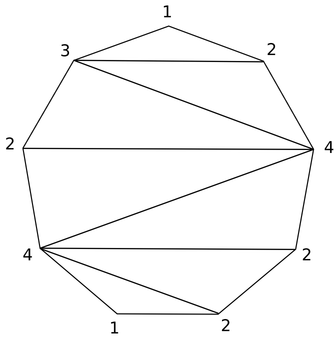

Fig. 1. Triangulation with quiddity cycle $(1, 3, 2, 4, 1, 2, 2, 4, 2)$.

Theorem 1.1 *(Theorem 2.10). For any $\ell \in \mathbb{N}$ we may compute finite sets of sequences E and F, where the elements of F have length at least ℓ, and such that every quiddity cycle not in E has an element of F as a (consecutive) subsequence.*

In other words, this theorem gives a local description of quiddity cycles. For example if $\ell = 4$:

Corollary 1.2. *Every quiddity cycle*[1] $c \notin \{(0,0),(1,1,1),(1,2,1,2)\}$ *contains at least one of*

$$\begin{gathered}(1,2,2,1),(1,2,2,2),(1,2,2,3),(1,2,2,4),(1,2,3,1),(1,2,3,2),\\(1,2,3,3),(1,2,4,1),(1,2,4,3),(1,2,5,1),(1,2,5,2),(1,2,6,1),\\(1,3,1,3),(1,3,1,4),(1,3,1,5),(1,3,1,6),(1,3,4,1),(1,4,1,2),\\(1,5,1,2),(1,6,1,2),(1,7,1,2),(2,1,3,2),(2,1,3,3),(2,2,1,4),\\(2,2,1,5),(3,1,2,3),(3,1,2,4).\end{gathered}$$

For instance, the cycle in Fig. 1 contains $(1, 2, 2, 4)$.

As an application, we reproduce the classification of [13][2]: Nichols algebras of diagonal type are straight forward generalizations of certain algebras investigated by Lusztig in the classical theory. They also produce a rich set of combinatorial invariants called the Weyl groupoids. The Weyl groupoid (see for example [10],[5]) was the essential tool in Heckenberger's classification of finite dimensional Nichols algebras of diagonal type in which case the real roots of the Weyl groupoid form a finite root system [11].

[1] Quiddity cycles are considered up to the action of the dihedral group, for example $(1, 2, 3, 1, 2, 3) \equiv (2, 1, 3, 2, 1, 3)$.

[2] Notice that J. Wang's result also relies on our main theorem. In this paper we considerably shorten the proof (see Theorem 2.12) since Wang requires a computer whereas our computation is entirely performed by hand.