Products of elementary matrices and non-Euclidean principal ideal domains

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Abstract

A classical problem, originated by Cohn's 1966 paper [1], is to characterize the integral domains R satisfying the property: (GE_n) "every invertible $n\times n$ matrix with entries in R is a product of elementary matrices". Cohn called these rings generalized Euclidean, since the classical Euclidean rings do satisfy (GE_n) for every $n > 0$. Important results on algebraic number fields motivated a natural conjecture: a non-Euclidean principal ideal domain R does not satisfy (GE_n) for some $n > 0$. We verify this conjecture for two important classes of non-Euclidean principal ideal domains: (1) the coordinate rings of special algebraic curves, among them the elliptic curves having only one rational point; (2) the non-Euclidean PID's constructed by a fixed procedure, described in Anderson's 1988 paper [2].

Keywords: non-Euclidean PID, generalized Euclidean rings, elementary matrices, idempotent matrices
2010 MSC: 15A23, 13F07, 14H05

Introduction

A main motivation for the present paper comes from two classical problems on factorizations of square matrices over rings.

The first one is to characterize the integral domains R that satisfy the following property

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