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# Products of elementary matrices and non-Euclidean principal ideal domains.

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## Abstract

A classical problem, originated by Cohn’s 1966 paper [1], is to characterize the integral domains  $R$  satisfying the property:  $(GE_n)$  “every invertible  $n \times n$  matrix with entries in  $R$  is a product of elementary matrices”. Cohn called these rings generalized Euclidean, since the classical Euclidean rings do satisfy  $(GE_n)$  for every  $n > 0$ . Important results on algebraic number fields motivated a natural conjecture: a non-Euclidean principal ideal domain  $R$  does not satisfy  $(GE_n)$  for some  $n > 0$ . We verify this conjecture for two important classes of non-Euclidean principal ideal domains: (1) the coordinate rings of special algebraic curves, among them the elliptic curves having only one rational point; (2) the non-Euclidean PID’s constructed by a fixed procedure, described in Anderson’s 1988 paper [2].

*Keywords:* non-Euclidean PID, generalized Euclidean rings, elementary matrices, idempotent matrices

*2010 MSC:* 15A23, 13F07, 14H05

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## Introduction

A main motivation for the present paper comes from two classical problems on factorizations of square matrices over rings.

The first one is to characterize the integral domains  $R$  that satisfy the following property

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