Journal of Algebra 501 (2018) 206–214

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

On the existence of birational surjective parametrizations of affine surfaces

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ARTICLE INFO

Article history:
Received 5 June 2017
Available online 21 February 2018
Communicated by Seth Sullivant

MSC:
primary 14Q10, 68W30

Keywords:
Rational surface
Birational parametrization
Surjective parametrization

ABSTRACT

In this paper we show that not all affine rational complex surfaces can be parametrized birational and surjectively. For this purpose, we prove that, if S is an affine complex surface whose projective closure is smooth, a necessary condition for S to admit a birational surjective parametrization from an open subset of the affine complex plane is that the curve at infinity of S must contain at least one rational component. As a consequence of this result we provide examples of affine rational surfaces that do not admit birational surjective parametrizations.

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1. Introduction

Some computational problems, of mathematical nature, can be approached by means of algebro-geometric techniques. In these situations, either because of the problem itself

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https://doi.org/10.1016/j.jalgebra.2017.12.028
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directly relates to an algebraic variety or because the problem is translated into an underlying algebraic variety, techniques from computational algebraic geometry are applied. Specially important are those cases where the associated algebraic variety is unirational, since then two different types of representations of the geometric object, namely a set of generators of its ideal or a rational parametrization of it, are available. Examples of this claim appear in some practical applications in computer geometric design (see [2], [5], [11], [16]), where the connection to algebraic geometry is direct. Other examples can be found in the study and solution of algebraic differential equations by means of the analysis of an associated algebraic variety (see e.g. [6], [8], [9], [12], [13], [14]); for instance, an algebraic non-autonomous first order ordinary differential equation induces an algebraic surface and the existence, and actual computation, of a general rational solution is derived from a birational parametrization of this surface (see [14]).

Nevertheless, when dealing with parametric representations one needs to guarantee that certain problematic situations do not appear. An specially important difficulty may occur when dealing with parametrizations that are not surjective. That is, let us work with, say, a rational affine variety X, and we take a birational affine parametrization f of X; in other words, a dominant birational map $f : \mathbb{C}^r \dashrightarrow f(\mathbb{C}^r) \subset X \subset \mathbb{C}^n$, and let us assume that f is not surjective, i.e. $f(\mathbb{C}^r) \subsetneq X$. Then, the feasibility of the use of f depends on whether the desired property of the variety, or the information derived from the variety, is only readable from the non-reachable zone $X \setminus f(\mathbb{C}^r)$ of the algebraic variety. Example 1.1., in [20], illustrates the described difficulty for the problem of computing the distance of a point to an algebraic surface. Another example of this situation can be found in Example 1, in [19], for computing the intersection of two surfaces using the implicit equation of one of the surfaces, and a parametrization of the other. In [18], Example 1, the authors illustrate the problem of analyzing cross sections in a surface when using a non surjective parametrization. An slightly different version of this commented difficulty can be found in the frame of algebraic differential solutions. Let us say that $y(x, c)$ is the rational general solution of an algebraic non-autonomous first order differential equation $F(x, y, y') = 0$. Then, $\mathcal{P}(x, c) = (x, y(x, c), y'(x, c))$, where the derivative is with respect to x, is a rational surface parametrization of the affine surface $F(x_1, x_2, x_3) = 0$. So, for each specialization of c, $\mathcal{P}(x, c)$ defines an integral rational curve on the surface. Now, observe that the non-surjectivity of $\mathcal{P}$ relates to the existence of rational singular solutions; and example illustrating this phenomenon can be found in [14], Example 7.1.

When the affine complex variety X is a curve, the problem admits a direct solution, in the sense that X can always be parametrized birationally and surjectively. Furthermore, in [1] and [17] one may find algorithms for this purpose. For the case of surfaces, the problem turns to be more complicated. The question has been approached from two different point of views: either providing one surjective birational affine parametrization of X (see e.g. [7], [15], [19]), or determining finitely many birational affine parametrizations $f_1, \ldots, f_s$ of X such that the union of their imagines does cover the whole affine

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