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# SILTING AND COSILTING CLASSES IN DERIVED CATEGORIES

FREDERIK MARKS, JORGE VITÓRIA

ABSTRACT. An important result in tilting theory states that a class of modules over a ring is a tilting class if and only if it is the Ext-orthogonal class to a set of compact modules of bounded projective dimension. Moreover, cotilting classes are precisely the resolving and definable subcategories of the module category whose Ext-orthogonal class has bounded injective dimension.

In this article, we prove a derived counterpart of the statements above in the context of silting theory. Silting and cosilting complexes in the derived category of a ring generalise tilting and cotilting modules. They give rise to subcategories of the derived category, called silting and cosilting classes, which are part of both a t-structure and a co-t-structure. We characterise these subcategories: silting classes are precisely those which are intermediate and Ext-orthogonal classes to a set of compact objects, and cosilting classes are precisely the cosuspended, definable and co-intermediate subcategories of the derived category.

## 1. INTRODUCTION

Silting and cosilting complexes, as introduced in [6], [27] and [28], can be understood as derived analogues of tilting and cotilting modules. This paper reinforces such a perspective through the torsion pairs naturally associated to these complexes, namely t-structures and co-t-structures. To every silting or cosilting complex we associate both a t-structure and a co-t-structure which turn out to be adjacent, that is, the *torsion* class of one of the pairs turns out to be the *torsionfree* class of the other. Such triples of subcategories are instances of so-called torsion-torsionfree (TTF) triples and the subcategory that links the two pairs will be referred to as the corresponding silting, respectively cosilting, class. For silting complexes, these triples were first observed in [21] in the bounded derived category of a finite-dimensional algebra of finite global dimension, and they were further generalised in [6] to unbounded derived categories of rings. For cosilting complexes, we will prove in this paper the existence of such triples, using results from [7], [16] and [28].

Our main aim is to describe silting and cosilting classes in the derived category of a ring from a recurrent point of view in (classical) tilting theory. In a module category it is well known that a class of modules is:

- a tilting class if and only if it is the right Ext-orthogonal class to a set of compact modules of bounded projective dimension (see [12] and the references therein);
- a cotilting class if and only if it is resolving, definable and such that the right Ext-orthogonal class to it has bounded injective dimension (see [11, Theorem 6.1]).

The first statement is known as the finite-type characterisation of tilting classes and it can be restated by saying that there is a bijection between resolving subcategories of compact modules of bounded projective dimension and tilting modules up to equivalence (see also [4]). Note that cotilting modules are generally not analogously determined by a set of compact modules. They are, however, always pure-injective ([9], [26]).

Our main theorem generalises the results above to (co)silting classes in the derived category of a ring.

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