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Presentations of topological full groups by generators and relations

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ABSTRACT

We describe generators and defining relations for the commutator subgroup of topological full groups of minimal subshifts. We show that the word problem in a topological full group is solvable if and only if the language of the underlying subshift is recursive.

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1. Introduction

Topological full groups (abbreviated as TFGs) first appeared in the theory of crossed-product C^*-algebras. A TFG can be defined as the group of automorphisms of a crossed product C^*-algebra preserving the maximal Abelian subalgebra modulo its center

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[16, Lemma 5.1 and Theorem 5.2], see also [17, Theorem 1]. The TFGs were proven to be complete invariants of the restricted isomorphism class of the crossed-product C^*-algebra [17, Theorem 2].

The topological full groups also play a major role in the classification theory of symbolic dynamical systems. It turns out that two minimal dynamical systems are flip conjugate (recall that two dynamical systems are called *flip conjugate* if they are conjugate or if one is conjugate to the inverse of the other) if and only if the associated TFGs are isomorphic as abstract groups [9,3]. We note that the term "topological" in TFG has been historically used to refer to the fact that these groups are associated with topological dynamical systems. No group topology is assumed on TFGs.

Recently, the construction of topological full groups and the interplay between their algebraic properties and the dynamical properties of underlying symbolic systems were used to establish the existence of *infinite finitely generated simple amenable groups*. It turns out that the commutator subgroups of TFGs associated with minimal subshifts over finite alphabets have the desired characteristics [11]. The discussion of algebraic properties of full groups can be found in [2,9,10,12–14].

The goal of the paper is to find a presentation of topological full groups and relate it to properties of the underlying dynamical system. Theorem 1.1 is the main result of the paper. It describes the set of defining relations and identifies groups with solvable word problem. The result is established in Theorem 4.1, Theorem 4.6, and Theorem 4.7.

Let (Ω, T) be a minimal subshift over a finite alphabet. Denote by G_T the topological full group of (Ω, T) and by G_T' the commutator subgroup of G_T (Definition 2.1). Denote by $L_n(\Omega)$ the set of words of length n appearing in sequences of Ω, $n \geq 1$. The language of the subshift is defined as $L(\Omega) = \bigcup_{n\geq 1} L_n(\Omega)$. The base of the topology on Ω comprises cylinder sets $(v, i) = \{\omega \in \Omega : \omega_{-i} = v_0, \ldots, \omega_{|v|-i} = v_{|v|-1}\}$, where $v \in L(\Omega)$, $i \in \mathbb{Z}$, and $|v|$ is the length of the word v. By a *cylinder partition* of (v, i), we mean a partition into cylinder sets, see Definition 3.2. In the following result, we use symbols $x_{(v,i)}$, $(v, i) \in L(\Omega) \times \mathbb{Z}$, as a base for the free group.

Theorem 1.1. *Let (Ω, T) be a minimal subshift over a finite alphabet. (1) There exists $n \geq 3$ such that the commutator subgroup of the topological full group G_T' is isomorphic to the group Γ_Ω generated by*

$$< x_{(w,k)}, w \in L(\Omega),\ |w| \geq n,\ k \in \mathbb{Z} >,$$

subject to the following relations: for every $w, v \in L(\Omega)$, $|w|, |v| \geq n$, $i, j \in \mathbb{Z}$, and a cylinder partition C of (w, i),

$$\left(x_{(w,i)}\right)^3 = 1 \tag{1}$$

$$\left(x_{(w,i)} \cdot x_{(w,i+1)}\right)^2 = 1 \tag{2}$$

$$x_{(w,i+1)} = x_{(w,i+2)} x^{-1}_{(w,i)} x^{-1}_{(w,i+2)} x_{(w,i)} \tag{3}$$

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