ARTICLE IN PRESS YJABR:16127

Journal of Algebra ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Graded identities of simple real graded division algebras

Yuri Bahturin [a,*,1], Diogo Diniz [b,2]

[a] *Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, NF, A1C5S7, Canada*
[b] *Unidade Acadêmica de Matemática, Universidade Federal de Campina Grande, Campina Grande, PB, 58429-970, Brazil*

ARTICLE INFO

Article history:
Received 13 December 2016
Available online xxxx
Communicated by N. Andruskiewitsch, A. Elduque, E. Khukhro and I. Shestakov

Keywords:
Simple real algebras
Graded algebras
Polynomial identities in algebras

ABSTRACT

Let A and B be finite dimensional simple real algebras with division gradings by an abelian group G. In this paper we give necessary and sufficient conditions for the coincidence of the graded identities of A and B. We also prove that every finite dimensional simple real algebra with a G-grading satisfies the same graded identities as a matrix algebra over an algebra D with a division grading that is either a regular grading or a non-regular Pauli grading. Moreover we determine when the graded identities of two such algebras coincide. For graded simple algebras over an algebraically closed field it is known that two algebras satisfy the same graded identities if and only if they are isomorphic as graded algebras.

* Corresponding author.
E-mail addresses: bahturin@mun.ca (Y. Bahturin), diogo@mat.ufcg.edu.br (D. Diniz).
[1] The first author acknowledges the support by an NSERC grant # 227060-14. The second author acknowledges the support by CAPES grant # 99999.001558/2014-05 and by CNPq grant # 303534/2013-3.
[2] The second author acknowledges the support of Atlantic Algebra Centre of Memorial University of Newfoundland where he was a Postdoctoral Fellow in January–September 2015.

http://dx.doi.org/10.1016/j.jalgebra.2017.02.016
0021-8693/© 2017 Elsevier Inc. All rights reserved.

1. Introduction

Two finite dimensional simple algebras over an algebraically closed field are isomorphic if and only if they satisfy the same polynomial identities. This is a direct consequence of Amitsur–Levitzki theorem. Analogous results have been proved for Lie algebras by A. Kushkulei and Yu. Razmyslov [10], for Jordan algebras by V. Drensky and M. Racine [8] and by E. Neher [11] and for more general non-associative algebras by I. Shestakov and M. Zaicev [15]. The analogous result for graded algebras was proved by P. Koshlukov and M. Zaicev [12] in the case of algebras graded by abelian groups and by E. Aljadeff and D. Haile [2] for arbitrary groups.

If the field is not algebraically closed, there exits non-isomorphic algebras that satisfy the same identities. For example, the real quaternion algebra and the algebra of 2×2 matrices satisfy the same polynomial identities. The goal of this paper is to precisely determine, when two finite-dimensional simple algebras over the field of real numbers graded by G, satisfy the same G-graded identities, provided that G is a finite abelian group.

For algebras over an algebraically closed field, the division gradings are regular (see Definition 1) and can be described in terms of a non-singular skew-symmetric bicharacter on the grading group (see [9, Theorem 2.38]). As it turns out, the graded polynomial identities for regular gradings are also determined by the corresponding bicharacter. Over the field of real numbers the situation is different. The division gradings for finite dimensional simple algebras over $\mathbb{R}$ have been classified in [4] and [14]. In this classification, distinct regular gradings arise that have the same associated bicharacter and thus non-isomorphic algebras may have the same graded identities. Each division grading is obtained either by restriction of scalars in a complex algebra or as a tensor product of a non-regular and a regular component. In our main result, Theorem 2, we prove, roughly speaking, that the polynomial identities in each case are determined by the bicharacter of the regular component. In the last section we obtain similar results for finite dimensional simple real algebras. We prove in Theorem 3 that every finite dimensional simple real algebra with a G-grading satisfies the same graded identities as a matrix algebra over an algebra D with a division grading that is either a regular grading or a Pauli grading. Then, in Theorem 4 we give necessary and sufficient conditions under which the graded identities of two such algebras coincide.

2. Preliminaries

We consider vector spaces, algebras, tensor products, etc., over the field $\mathbb{R}$ of the real numbers. Let G be a group with identity element e and A an algebra. A vector space decomposition $A = \oplus_{g \in G} A_g$ is said to be a G**-grading** if $A_g A_h \subseteq A_{gh}$ for every $g, h \in G$. The **support** of the grading, denoted by supp A, is the set supp $A = \{g \in G | A_g \neq 0\}$. If $A = A_e$ we say that the grading is **trivial**. An ideal I of A is called **homogeneous** if $I = \oplus_{g \in G}(I \cap A_g)$ and A is **graded simple** if 0 and A are the only homogeneous ideals of A.