ARTICLE IN PRESS YJABR:16145

Journal of Algebra ••• (••••) •••–•••

Contents lists available at ScienceDirect

Journal of Algebra

www.elsevier.com/locate/jalgebra

Structure of parabolically induced modules for affine Kac–Moody algebras

Vyacheslav Futorny *, Iryna Kashuba

Institute of Mathematics, University of São Paulo, Caixa Postal 66281, CEP 05314-970, São Paulo, Brazil

ARTICLE INFO

Article history:
Received 30 September 2016
Available online xxxx
Communicated by N. Andruskiewitsch, A. Elduque, E. Khukhro and I. Shestakov

Dedicated to Efim Zelmanov on the occasion of his 60th birthday

Keywords:
Affine Lie algebra
Weight module
Induced module
Parabolic subalgebra

ABSTRACT

The main result of the paper establishes the irreducibility of a large family of nonzero central charge induced modules over Affine Lie algebras for any non standard parabolic subalgebra. It generalizes all previously known partial results and provides a construction of many new irreducible modules.

1. Introduction

Let $\mathfrak{G}$ be an affine Kac–Moody algebra with a 1-dimensional center $Z = \mathbb{C}c$ and a fixed Cartan subalgebra.

The main problem in the representation theory of affine Kac–Moody algebras is a classification of all irreducible weight representations. Such classification is known in various subcategories of weight modules, e.g. in the category $\mathcal{O}$, in its generalizations

* Corresponding author.
E-mail addresses: futorny@ime.usp.br (V. Futorny), kashuba@ime.usp.br (I. Kashuba).

http://dx.doi.org/10.1016/j.jalgebra.2017.03.007
0021-8693/© 2017 Published by Elsevier Inc.

[6,3,12], in the category of modules with finite dimensional weight multiplicities and nonzero central charge [13]. An important tool in the construction of representations of affine Lie algebras is a parabolic induction. The conjecture ([7], Conjecture 8.1) indicates that induced modules are construction devices for irreducible weight modules. This conjecture is known to be true for $A_1^{(1)}$ ([9], Proposition 6.3), for $A_2^{(2)}$, [2] and for all affine Lie algebras in the case of modules with finite-dimensional weight spaces [13,4].

Simplest case of parabolic induction corresponds to the induction from Borel subalgebras. Standard examples of Borel subalgebras arise from taking partitions of the root system. For affine algebras there is always a finite number of conjugacy classes by the Weyl group of such partitions and corresponding Borel subalgebras. Verma type modules induced from these Borel subalgebras were first studied and classified by Jakobsen and Kac [14,15], and by Futorny [6,8], and were further developed in [3,12,7,9] and references therein. We will consider a more general definition of a Borel subalgebra (see below).

Nontrivial (different from Borel) parabolic subalgebras are divided into two groups, those with finite dimensional Levi subalgebras and those with infinite dimensional one. In this paper we are interested in the second case in this paper. The simplest non trivial example is given by a parabolic subalgebra whose Levi factor is the Heisenberg subalgebra together with Cartan subalgebra. Corresponding induced modules were studied in recent papers [11] and [1]. It was shown that any irreducible $\mathbb{Z}$-graded module over the Heisenberg subalgebra with a nonzero central charge induces the irreducible $\mathfrak{G}$-module. In [10] a similar reduction theorem was shown for pseudo parabolic subalgebras. These parabolic subalgebras give a particular class of non-solvable parabolic subalgebra of $\mathfrak{G}$ with infinite dimensional Levi factor. The main results of [10] states that in this case the parabolic induction preserves irreducibility if the central charge is nonzero. The technique used in the proofs in [10] and [11] are different and somewhat complimentary.

The main purpose of the present paper is to show that in the affine setting both these cases of parabolic induction (and hence all known cases) can be extended to a more general result for modules with nonzero central charge.

For any Lie algebra $\mathfrak{a}$ we denote by $U(\mathfrak{a})$ the universal enveloping algebra of $\mathfrak{a}$.

Denote by G the Heisenberg subalgebra of $\mathfrak{G}$ generated by all imaginary root subspaces of $\mathfrak{G}$. Let $\mathcal{P} \subset \mathfrak{G}$ be a parabolic subalgebra of $\mathfrak{G}$ such that $\mathcal{P} = \mathfrak{l} \oplus \mathfrak{n}$ is a Levi decomposition and $\mathfrak{l}$ is infinite dimensional Levi factor. Denote by $\mathfrak{l}^0$ the Lie subalgebra of $\mathfrak{l}$ generated by all its real root subspaces and $\mathfrak{H}$. Let $G(\mathfrak{l})$ be a subalgebra of $\mathfrak{l}^0$ spanned by its imaginary root subspaces. Then $\mathfrak{l} = \mathfrak{l}^0 + G_{\mathfrak{l}}$ where $G_{\mathfrak{l}} \subset G$ is the orthogonal complement of $G(\mathfrak{l})$ in G with respect to the Killing form, that is $G = G(\mathfrak{l}) + G_{\mathfrak{l}}$, $[G_{\mathfrak{l}}, \mathfrak{l}^0] = 0$ and $\mathfrak{l}^0 \cap G_{\mathfrak{l}} = \mathbb{C}c$.

For a Lie algebra $\mathfrak{a}$ containing the Cartan subalgebra $\mathfrak{H}$ we say that a module V is a *weight* module if $V = \oplus_{\mu \in \mathfrak{H}^*} V_\mu$, where

$$V_\mu = \{v \in V | hv = \mu(h)v, \forall h \in \mathfrak{H}\}.$$

Please cite this article in press as: V. Futorny, I. Kashuba, Structure of parabolically induced modules for affine Kac–Moody algebras, J. Algebra (2017), http://dx.doi.org/10.1016/j.jalgebra.2017.03.007