Accepted Manuscript

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PII: S0021-8693(17)30261-2
DOI: http://dx.doi.org/10.1016/j.jalgebra.2017.04.020
Reference: YJABR 16208

To appear in: *Journal of Algebra*

Received date: 13 September 2016

Simple finite-dimensional double algebras

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Abstract

A double algebra is a linear space V equipped with linear map $V \otimes V \to V \otimes V$. Additional conditions on this map lead to the notions of Lie and associative double algebras. We prove that simple finite-dimensional Lie double algebras do not exist over an arbitrary field, and all simple finite-dimensional associative double algebras over an algebraically closed field are trivial. Over an arbitrary field, every simple finite-dimensional associative double algebra is commutative. A double algebra structure on a finite-dimensional space V is naturally described by a linear operator R on the algebra $\operatorname{End} V$ of linear transformations of V. Double Lie algebras correspond in this sense to skew-symmetric Rota—Baxter operators, double associative algebra structures—to (left) averaging operators.

Keywords: double Lie algebra, Rota—Baxter operator, averaging operator
2000 MSC: 16K20, 15A24, 16W99

1. Introduction

The general philosophy of noncommutative geometry which goes back to M. Kontsevich states that a noncommutative geometric structure on an associative algebra A should turn into an ordinary geometric structure on the variety of n-dimensional representations of A under the functor Rep_n from the category of associative algebras to the category of schemes. In