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Weyl groups of some hyperbolic Kac–Moody algebras

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A B S T R A C T

We use the theory of Clifford algebras and Vahlen groups to study Weyl groups of hyperbolic Kac–Moody algebras T_n^{++}, obtained by a process of double extension from a Cartan matrix of finite type T_n, whose corresponding generalized Cartan matrices are symmetric.

Contents

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1. Introduction

In [11], Feingold and Frenkel gained significant new insight into the structure of a particularly interesting rank 3 hyperbolic Kac–Moody algebra which they called $\mathcal{F}$ (also known as A_1^{++}), along with some connections to the theory of Siegel modular forms of genus 2. The first vital step in their work was the discovery that the even part of the Weyl group of that Kac–Moody algebra is $\mathcal{SW}(\mathcal{F}) \cong PSL(2,\mathbb{Z})$. (If $\mathcal{W}$ is a Weyl group, we will denote its even part by $\mathcal{SW}$.)

In [12], a coherent picture of Weyl groups was presented for many higher rank hyperbolic Kac–Moody algebras using lattices and subrings of the four normed division algebras. Specifically, the Weyl groups of all hyperbolic algebras of ranks 4, 6 and 10 which can be obtained by a process of double extension, admit realizations in terms of generalized modular groups over the complex numbers $\mathbb{C}$, the quaternions $\mathbb{H}$, and the octonions $\mathbb{O}$, respectively. In particular, the authors found in the rank four situation that the even part of the Weyl group of the Kac–Moody algebra A_2^{++} is the Bianchi group $PSL(2, O_{-3})$, where O_{-3} is the ring of integers of $\mathbb{Q}(\sqrt{-3})$.

One could ask if there is a similar phenomenon for all the hyperbolic Kac–Moody algebras T_n^{++}, where T_n is any finite type root system, but it is not clear what to take instead of a normed division algebra. In [14], the authors used the quaternions and the octonions in their study of some Weyl groups. In this paper, we adopt another approach. We use the theory of Vahlen groups and Clifford algebras in order to study the Weyl groups of the hyperbolic Kac–Moody algebras T_n^{++} whose Cartan matrices are symmetric. A key ingredient needed to obtain our main result in Theorem 6.5 is