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Invariants of maximal tori and unipotent constituents of some quasi-projective characters for finite classical groups

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ABSTRACT

We study the decomposition of certain reducible characters of classical groups as the sum of irreducible ones. Let $\mathbf{G}$ be an algebraic group of classical type with defining characteristic $p > 0$, μ a dominant weight and W the Weyl group of $\mathbf{G}$. Let $G = G(q)$ be a finite classical group, where q is a p-power. For a weight μ of $\mathbf{G}$ the sum s_μ of distinct weights $w(\mu)$ with $w \in W$ viewed as a function on the semisimple elements of G is known to be a generalized Brauer character of G called an orbit character of G. We compute, for certain orbit characters and every maximal torus T of G, the multiplicity of the trivial character 1_T of T in s_μ. The main case is where $\mu = (q-1)\omega$ and ω is a fundamental weight of $\mathbf{G}$. Let St denote the Steinberg character of G. Then we determine the unipotent characters occurring as constituents of $s_\mu \cdot St$ defined to be 0 at the p-singular elements of G. Let β_μ denote the Brauer character of a representation of $SL_n(q)$ arising from an irreducible representation of $\mathbf{G}$ with highest weight μ. Then we determine the unipotent constituents of the characters $\beta_\mu \cdot St$ for $\mu = (q-1)\omega$, and also for some other μ (called strongly q-restricted). In addition, for strongly

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restricted weights μ, we compute the multiplicity of 1_T in the restriction $\beta_\mu|_T$ for every maximal torus T of G.

1. Introduction

The groups G under consideration in this paper are $GL_n(q)$, $SL_{n+1}(q)$, $Sp_{2n}(q)$, $SO_{2n+1}(q)$, q odd, $SO^{\pm}_{2n}(q)$, q odd, $Spin^{\pm}_{2n}(q)$, q even. Let $\overline{F}_q$ be the algebraic closure of a finite field F_q of q elements. Let $\mathbf{G}$ be the respective algebraic group over $\overline{F}_q$, and W the Weyl group of $\mathbf{G}$. For the notion of a maximal torus in G see [9,4]. The maximal tori of G, up to G-conjugation, are in bijection with the conjugacy classes of W unless $G = SO^-_{2n}(q)$, q odd, and $Spin^-_{2n}(q)$, q even [4, 3.3.3]. So we denote by T_w any maximal torus of G from the class corresponding to $w \in W$.

Let $\mathbf{T}$ be the group of diagonal matrices in $GL_n(\overline{F}_q)$. Let ε_i be the mapping sending every diagonal matrix $\mathrm{diag}(x_1, \dots, x_n)$ to the i-th entry x_i $(1 \leq i \leq n)$. There is a natural embedding $GL_n(\overline{F}_q) \to \mathbf{G}$ which identifies $\mathbf{T}$ with a maximal torus of $\mathbf{G}$. So $\varepsilon_1, \dots, \varepsilon_n$ can be viewed as weights of $\mathbf{G}$, as well as $\sum a_i\varepsilon_i$ for $a_i \in \mathbb{Z}$. Set $\omega_i = \varepsilon_1 + \cdots + \varepsilon_i$ $(1 \leq i \leq n)$. Then ω_i is a fundamental weight of $\mathbf{G}$, unless $i = n$ for $\mathbf{G}$ of type B_n and $i = n-1, n$ for type D_n. As W acts on the weights of $\mathbf{G}$, we may set $W_i = \{w \in W : w(\omega_i) = \omega_i\}$. It is well known that W_i is the Weyl group of a certain Levi subgroup $\mathbf{L}_i$ of $\mathbf{G}$. For finite groups $A \subset B$ denote by 1_A the trivial character of A and by 1_A^B the induced character. For a weight μ of $\mathbf{G}$ let s_μ denote the respective orbit character. Note that for a maximal torus T of G the restriction $s_\mu|_T$ is an ordinary character of T, and $(s_\mu|T, 1_T)$ denotes the inner product of the characters $s_\mu|_T$ and the trivial character 1_T of T.

Theorem 1.1. *Let $\mathbf{G}, G, W$ be as above, $G \neq SO^-_{2n}(q), Spin^-_{2n}(q)$, and for $w \in W$ let T_w be a respective maximal torus in G. Let $\mu = (q-1)\omega_i$, where $i \in \{1, \dots, n\}$, and s_μ be the respective orbit character. Then $(s_\mu|_{T_w}, 1_{T_w}) = 1^W_{W_i}(w)$.*

Unipotent characters are introduced by Deligne and Lusztig [8]. For any character σ of G we denote by $u(\sigma)$ the "unipotent part" of σ, which is the sum of all unipotent irreducible constituents of σ regarding multiplicities. For the notions of Harish-Chandra induction and the Steinberg character see [9, Ch. 4, 9]. If τ is a character of a Levi subgroup L of G then $\tau^{\#G}$ denotes the Harish-Chandra induced character.

Theorem 1.2. *Let $\mathbf{G}, G, W, \omega_i, W_i$ be as above, $\mu = (q-1)\omega_i$ $(1 \leq i \leq n)$ and let s_μ be the orbit character of G corresponding to μ. If $G = SO^-_{2n}(q)$ or $Spin^-_{2n}(q)$, assume $j < n$. Then $u(s_\mu \cdot St) = St^{\#G}_{L_i}$, where L_i is a Levi subgroup of G with Weyl group W_i and St_i the Steinberg character of L_i.*

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