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Classification of finite irreducible conformal modules over a class of Lie conformal algebras of Block type

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ABSTRACT

We classify finite irreducible conformal modules over a class of infinite Lie conformal algebras $\mathfrak{B}(p)$ of Block type, where p is a nonzero complex number. In particular, we obtain that a finite irreducible conformal module over $\mathfrak{B}(p)$ may be a nontrivial extension of a finite conformal module over $\mathfrak{Vir}$ if $p = -1$, where $\mathfrak{Vir}$ is a Virasoro conformal subalgebra of $\mathfrak{B}(p)$. As a byproduct, we also obtain the classification of finite irreducible conformal modules over a series of finite Lie conformal algebras $\mathfrak{b}(n)$ for $n \geq 1$.

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1. Introduction

Lie conformal algebras, introduced by Kac [17], encode the singular part of the operator product expansion of chiral fields in conformal field theory. The theory of finite Lie conformal algebras has been greatly developed in the last two decades (e.g., [2,7–9,11,17,18,28]). Finite simple Lie conformal algebras were classified in [9], which shows that a finite simple Lie conformal algebra is isomorphic to either the Virasoro conformal algebra or a current conformal algebra $\mathrm{Cur}\,\mathfrak{g}$ over a simple finite-dimensional Lie algebra $\mathfrak{g}$. The theory of conformal modules and their extensions was developed in [7,8], and the cohomology theory was developed in [2] and further in [11]. For super cases, the structure and representation theories have also been developed in recent years, see [14,15,5,20] and the references therein.

However, the theory of infinite Lie conformal algebras is far from being well developed. The most important example is the general Lie conformal algebra gc_N, which plays the same role in the theory of Lie conformal algebras as the general Lie algebra gl_N does in the theory of Lie algebras. The important difference between usual and conformal algebras is that gc_N is infinite. Due to these reasons, the general Lie conformal algebra gc_N and its subalgebras have been studied by many authors (e.g., [3,4,6,9,10,21,25,29]). Recently, some interesting examples of infinite Lie conformal algebras associated with infinite-dimensional loop Lie algebras were constructed and studied (e.g., [12,13,26]).

In this paper, we focus on another class of infinite Lie conformal algebras $\mathfrak{B}(p)$ with p being a nonzero complex number, where $\mathfrak{B}(p)$ has a $\mathbb{C}[\partial]$-basis $\{L_i \,|\, i \in \mathbb{Z}_+\}$ and λ-brackets

$$[L_i \,{}_\lambda\, L_j] = ((i+p)\partial + (i+j+2p)\lambda)L_{i+j}. \tag{1.1}$$

We refer to $\mathfrak{B}(p)$'s as *Lie conformal algebras of Block type* due to their relations with some Lie algebras of Block type (see Remark 2.7). There are some interesting features on this class of Lie conformal algebras. Firstly, each $\mathfrak{B}(p)$ contains a Virasoro conformal subalgebra. Set $L = \frac{1}{p}L_0 \in \mathfrak{B}(p)$. By (1.1), we see that $[L\,{}_\lambda\, L] = (\partial + 2\lambda)L$. Namely, the subalgebra

$$\mathfrak{Vir} = \mathbb{C}[\partial]L \tag{1.2}$$

of $\mathfrak{B}(p)$ is exactly the Virasoro conformal algebra. Secondly, the special case $\mathfrak{B}(1)$ has close relation with the general Lie conformal algebra gc_1. In fact, $\mathfrak{B}(1)$ is a maximal subalgebra of the associated graded conformal algebra $\mathrm{gr}\, gc_1$ of the filtered algebra gc_1 [25]. Thirdly, there are embedding relations among $\mathfrak{B}(p)$'s. For any integer $n \geq 1$, $\mathfrak{B}(p)$ can be embedded into $\mathfrak{B}(np)$ via $L_i \mapsto \frac{1}{n}L'_{ni}$. Finally, $\mathfrak{B}(-n)$ contains a series of finite Lie conformal quotient algebras (cf. (2.2))

$$\mathfrak{b}(n) = \mathfrak{B}(-n)/\mathfrak{B}(-n)_{\langle n+1\rangle},$$